

Contents

Introduction	vii
1 Basic Concepts	1
1.1 Algebras	1
1.2 Compatible relations of an algebra	2
1.3 Lattices	4
1.4 Varieties	5
2 Hyperidentities and Hypervarieties	11
2.1 Hypersubstitutions and hyperidentities	13
2.2 Hyperequational classes	17
2.3 Hyperequational logic	20
2.4 Hypervarieties	26
2.5 The clone of a variety	30
2.6 Hypervarieties of type τ	41
2.7 The lattice of all solid varieties of type τ	45
2.8 Free clones and solidifyable varieties	56
2.9 Hypervarieties and bi-identities	63
3 Hyperidentities in Varieties of Semigroups	69
3.1 Basic concepts	69
3.2 Hyperassociative varieties of semigroups	77
3.3 Solid varieties of semigroups	92
3.4 Other types of algebras	99
4 M-Hyperidentities and M-Solid Varieties	105
4.1 Monoids of hypersubstitutions	106
4.2 Conjugate pairs of additive closure operators	110
4.3 The lattices of M -solid varieties	117
4.4 V -proper hypersubstitutions and normal forms	118
4.5 Examples of monoids of hypersubstitutions	124
4.6 Subvarieties and submonoids	137

4.7 Relations on monoids of hypersubstitutions	141
4.8 Derived varieties and equational theories	144
4.9 Hyperquasi-identities and M -solid quasivarieties	152
4.10 M -solid pseudovarieties	157
5 M-Solid Varieties of Semigroups	167
5.1 Pre-solid varieties of semigroups	167
5.2 Edge-solid varieties of semigroups	181
5.3 Regular-solid and dual-solid varieties	188
5.4 Maps between M -solid varieties of semigroups	193
5.5 Algorithmic problems	198
5.6 Pre-solid and solid varieties of type $\tau = (n)$	202
6 Describing Clones by Hyperidentities	209
6.1 Basic ideas	209
6.2 The lattice of all clones on a finite set	212
6.3 The lattice of all Boolean clones	216
6.4 Characterizing hyperidentities	219
6.5 Separation of clones by hyperidentities	235
6.6 Separating hyperidentities for Boolean clones	238
6.7 Varieties generated by Boolean clones	242
6.8 Projection clones	243
6.9 Maximal clones and functional completeness	245
7 Hyperidentities in Partial Algebras	259
7.1 Terms and identities for partial algebras	260
7.2 Hypersubstitutions of type τ	264
7.3 Regular hypersubstitutions	266
7.4 The closure operators χ_M^E and χ_M^A	269
7.5 Characterization of primal partial algebras	274
8 Exercises and Problems	285
8.1 Exercises	285
8.2 Problems	287
Bibliography	293
Index	308