# Table of contents

Preface, vii

Chapter 1: Epistemology

#### 1.1 The purpose of this book, 1

The question of what to study, and to what end? Looking at *Mathematica* from the scientist's perspective. *Mathematica* programming references. NeXT programming references.

#### 1.2 Mathematica for education and research, 3

The "calculator crisis" turned around. NeXT window suite. Using *Mathematica* to prepare algorithms in view of fast machine attack.

#### 1.3 Book presentation, 5

The trend of chapters herein. Expression numbering and *Mathematica* program numbering. Format adopted for input/output.

# Chapter 2: NeXT interfaces and projects

## 2.1 Standard NextStep interface, 7

Operation of Release 1.0 *Mathematica* under NextStep. Typical application window appearance. UNIX and concurrency of applications under NextStep.

## 2.2 Example applications that talk Mathematica, 9

CircuitBuilder application generates pasteable circuit equations. ExpressionBuilder application features expression object embedded in PostScript output.

## 2.3 Automated messaging, 12

RealTimeAlgebra application uses NextStep Speaker/ Listener connection. Gourmet calculator application uses Droid object for interprocess communication. MathLink standard for the future discussed.

# Chapter 3: Graphics for the sciences

#### 3.1 The interplay of 2D and 3D graphics, 15

Gamma function as canonical example of 2D/3D interplay. Visualization and verification of Gamma function asymptotics.

#### 3.2 The importance of resolution, 19

Bad plot of the radial sinc() function: problem of spatial aliasing, cured by sufficient resolution.

#### 3.3 Insight via the contour option, 22

Helicoid surface shows ruled property. Branch cuts exhibited via Contour[] option.

#### 3.4 3D animation of rigid structures, 24

Animation of a rigid pyramidal shell. The importance of extra view orientations.

#### 3.5 When y is not a function of x, 26

Exponential spiral example. Newtonian elliptical orbit example.

## 3.6 3D parametric plots, 29

Trefoil knot in 2D, 3D, and "sausage" modes. Moebius band, one-sided character of which observed via DensityPlot[].

## 3.7 Further explorations, 38

## Chapter 4: Mathematical examples

## 4.1 Identities and expansions, 39

Generating function for Legendre polynomials. Rodrigues formula. Poisson summation identity. Jacobi theta function identity. Representations by two integer squares. Cyclotomic polynomials.

## 4.2 Real and complex analysis, 49

Exact evaluation of  $\zeta(4)$ . Proof that a surface is a minimal surface. Exact integrals via residue calculus.

## 4.3 Factorization and primality testing algorithms, 57

Pollard "rho" factorization. Pollard (p-1) factorization. Elliptic curve factorization. Rigorous primality testing. Lucas-Lehmer method for testing of Mersenne numbers.

#### 4.4 Fast algorithms, 67

Muliplication via FFT-based convolution. Fast polynomial reciprocation via Newton's method with adjusted precision. Pre-conditioned Chinese Remainder algorithm.

#### 4.4 Further explorations, 74

# Chapter 5: Physics

#### 5.1 Classical mechanics, 75

Hamiltonian formalism. Simple harmonic oscillator. True anharmonic pendulum. Symbolic Hamiltonian iteration. Euler and Runge-Kutta solvers. Pendulum period analyzed via elliptic integrals and AGM.

#### 5.2 Quantum mechanics, 88

Tunnel effect. Symbolic treatment of quantum oscillator. Quantum perturbation theory to 1st and 2nd orders. Numerical integration of the Schroedinger equation.

#### 5.3 Relativity, 107

Einstein energy expansion. Relativistic action principle. Perturbative and exact action methods. Parker-Christenson *MathTensor* system example output. Four-vector relativity calculus and Compton scattering of photons.

## 5.4 Further explorations, 126

# Chapter 6: Linear and non-linear systems

## 6.1 Linear oscillations, 127

Normal modes. Eigenvalues and eigenvectors. Membrane oscillations and Bessel functions. Damped oscillator. Resonance, phase, frequency shift due to damping.

## 6.2 Solitons, 140

Kortewig-de Vries equation. Pitfalls of naive numerical integration. Numerical improvements: double-time-step and proper 3rd difference operator. Space-time soliton collision diagrams. Exact KdV solutions via *Mathematica* symbolics.

## 6.3 Chaos and fractals, 156

Bifurcation in quadratic maps. Feigenbaum period-doubling

constant. Mandelbrot set. Sierpinski gasket fractal. Fractal nature of recursive squaring. Measurement of fractal dimensions.

#### 6.4 Further explorations, 168

# Chapter 7: Chemistry and biology

#### 7.1 Reactions, 169

Balancing of reactions and stoichiometry. Equilibrium reaction algebra. Efficient chemical production from expensive reagents.

#### 7.2 Quantum chemistry, 175

Hydrogen atom treatment via finite element method. Helium atom: exact variational treatment for separated trial wave function *ansatz*.

#### 7.3 Genetics and population biology, 185

Algebra for recombinant gene data. Population combinatorics for two alleles. Genetic drift and extinction. Exact treatment of Markov process time expectations.

## 7.4 Neurobiology, 197

Nerve action potentials. Hodgkin-Huxley equations. Explicit calculation of nerve propagation velocity.

## 7.5 Further explorations, 204

# Chapter 8: Electronics and signal processing

## 8.1 Electronic circuits, 205

LC tank circuit. Resonance and phase plots.

3rd order active filter example. Non-linear diode circuit. Logic gate circuit.

## 8.2 Applications of the FFT, 217

Relation between standard engineer's FFT and *Mathematica* Fourier[] and InverseFourier[]. Symmetry rules for pure-real FFTs. Square-wave spectrum. Tone burst spectrum. Windowing technique for reduction of sidebands. AM signal spectrum: decibel plot. Real-world speech signal

analyzed. Spectrogram and sonogram plots.

#### 8.3 Digital filters, 232

General LTI filter algorithm. Recursive, tunable bandpass example. Non-recursive filter design. Bandpass example with explicit center frequency and bandwidth input parameters. Example plot of signal pulled out of noise.

#### 8.4 Image processing, 241

Loading of image files. Pixel arithmetic in *Mathematica*. Edge detection via Laplacian operator.

8.5 Further explorations, 246

# Chapter 9: Great problems of history

#### 9.1 Problems solved and unsolved, 247

Great problems of history are solved, partially solved, or open. *Mathematica* explorations can verify and sometimes reproduce historical results.

## 9.2 Fermat's "Last Theorem", 248

Kummer's regular primes. The Vandiver criterion for irregular primes. Alternative Bernoulli number identities. Current status of numerical research.

## 9.3 The Riemann Zeta function and prime numbers, 255

Euler product for  $\zeta(s)$  verified. Prime Number Theorem stated. Rigorous results on  $\zeta$  zeros exemplified. Polynomial-root approximation of zeros via Hermite series introduced. Accurate estimates for  $\pi(x)$ . Zeros of  $\zeta(s)$  on Re(s) = 1.

## 9.4 Theories of gravitation, 270

Newton's original result: a sphere gravitates as if concentrated at its center. Derivation of Newtonian orbits from first principles. General-relativistic precession of the orbit of Mercury.

9.5 Further explorations, 281

References, 283

Index, 289