

Contents

31	4.3.5	Rectangular hybrid automata	2.7	Exercises	64
31	4.3.6	Timed automata			65
31	4.4	Semantics: Hybrid executions		Modeling Physics	65
31	4.4.1	Numerical simulation	3.1	Quick introduction to differential equations	67
31	4.4.2	Reachable states	3.1.1	Example: Vehicle speed control	68
31	4.4.3	Time	3.1.2	Language for specifying differential equations	69
33	4.4.4	Execution zoo	3.2	Specifying ordinary differential equations	71
34	4.5	Example: Spacecraft docking	3.2.1	State variables and valuations	72
34	4.6	Example: Small aircraft traffic	3.2.2	Dense time and trajectories	74
35	4.7	Exercises	3.2.3	Trajectories as solutions	75
37			3.3	Special classes of ODEs	75
37		Composing Models	3.3.1	Time-invariant and autonomous systems	79
38		Preface	3.3.2	Linear systems	xv
39	1	Introduction	3.4	Semantics: Reachable states, invariants, and stability	80
40	1.1	What is this book about? The verification problem	3.4.1	Example: Pendulum	80
42	1.2	Testing and verification for establishing system requirements	3.4.2	Example: Vehicle speed control	81
44	1.3	From systems to models of systems	3.5	Lyapunov's direct method for proving stability	82
45	1.4	Design automation ecosystem	3.5.1	Stability of linear dynamical systems	83
46	1.5	Challenges and state of the art	3.6	Differential equations as automata	88
47	1.6	Road map	3.7	Example: Simple economy	89
48	1.7	Learning and teaching using this book	3.8	Numerical simulations for control synthesis	90
49	2	Modeling Computation	3.9	Closing the loop and control synthesis	91
51	2.1	Quick introduction to automata	3.9.1	Proportional-integral-derivative controller	17
52	2.1.1	Example: JK flip-flop	3.9.2	Controller synthesis problem	17
53	2.1.2	Language for specifying automata	3.10	Exercises	17
55	2.2	Specifying automata		Modeling Cyber-Physical Systems	20
55	2.2.1	State variables and valuations	4.1	Quick introduction to hybrid automata	20
55	2.2.2	Predicates	4.1.1	Example: Rimless wheel	20
56	2.2.3	Transitions	4.1.2	Language for specifying hybrid systems	21
58	2.2.4	Automata	4.2	Specifying hybrid automata	22
58	2.3	Special automata classes	4.2.1	State variables and transitions	23
58	2.3.1	Finite and discrete automata	4.2.2	Trajectories and closures	23
60	2.3.2	Nondeterminism	4.2.3	Hybrid automata	23
61	2.3.3	Discrete sequences and sampled time	4.2.4	A guide for hybrid automata	23
61	2.4	Semantics: Executions, reachable states, and invariants	4.3	Special classes	24
61	2.5	Example: Dijkstra's token ring algorithm	4.3.1	Deterministic hybrid automata	24
62	2.5.1	Legal states and invariants	4.3.2	Switched systems	26
63	2.5.2	Asynchronous and synchronous models	4.3.3	Linear hybrid automata	26
63	2.6	Example: Reasoning about impossibility	4.3.4	Example: Thermodynamics	27

2.7	Exercises	28
3	Modeling Physics	31
3.1	Quick introduction to differential equations	31
3.1.1	Example: Vehicle speed control	31
3.1.2	Language for specifying differential equations	31
3.2	Specifying ordinary differential equations	33
3.2.1	State variables and valuations	34
3.2.2	Dense time and trajectories	34
3.2.3	Trajectories as solutions	35
3.3	Special classes of ODEs	37
3.3.1	Time-invariant and autonomous systems	37
3.3.2	Linear systems	38
3.4	Semantics: Reachable states, invariants, and stability	39
3.4.1	Example: Pendulum	40
3.4.2	Example: Kinematic vehicle model	42
3.5	Lyapunov's direct method for proving stability	44
3.5.1	Stability of linear dynamical systems	45
3.6	Differential equations as automata	46
3.7	Example: Simple economy	47
3.8	Numerical simulations for ordinary differential equations	48
3.9	Closing the loop and control synthesis	49
3.9.1	Proportional-integral-derivative controller	51
3.9.2	Controller synthesis problem	52
3.10	Exercises	53
4	Modeling Cyber-Physical Systems	55
4.1	Quick introduction to hybrid automata	55
4.1.1	Example: Rimless wheel	55
4.1.2	Language for specifying hybrid systems	56
4.2	Specifying hybrid automata	58
4.2.1	State variables and transitions	58
4.2.2	Trajectories and closures	58
4.2.3	Hybrid automata	60
4.2.4	A guide for hybrid modeling	61
4.3	Special classes of hybrid automata	61
4.3.1	Deterministic hybrid automata	61
4.3.2	Switched systems	62
4.3.3	Linear hybrid automata	63
4.3.4	Example: Thermostat	63

108	4.3.5	Rectangular hybrid automata	64
107	4.3.6	Timed automata	65
109	4.4	Semantics: Hybrid executions	65
110	4.4.1	Numerical simulation of hybrid executions	67
111	4.4.2	Reachable states, invariants, and stability	68
112	4.4.3	Time-abstract semantics	69
113	4.4.4	Execution zoo	71
113	4.5	Example: Spacecraft docking	72
116	4.6	Example: Small aircraft traffic management system	74
116	4.7	Exercises	75
111	5	Composing Models	79
117	5.1	Composing automata	79
117	5.2	Composing input/output automata	80
118	5.2.1	Input/output automata	80
118	5.2.2	Compatibility and composition of input/output automata	80
118	5.3	Example: Channels, logical clocks, and distributed systems	81
119	5.3.1	First-in, first-out channels	81
120	5.3.2	Logical time in distributed systems: Lamport clocks	83
121	5.3.3	Composed system: A network of processes communicating over channels	84
123	5.3.4	Traces and projections	85
123	5.4	Composing hybrid input/output automata	87
125	5.4.1	Hybrid input/output automata	88
127	5.4.2	Compatibility and composition of hybrid input/output automata	89
127	5.5	Example: Interconnecting flip-flops	90
130	5.6	Example: Timed channels	91
133	5.7	Example: Pulse generator and oscillator	93
133	5.8	Traces, untiming, and properties of compositions	93
134	5.9	Example: Emergency braking on highways	96
136	5.10	Exercises	98
137	6	Specifying Requirements	101
137	6.1	Requirements analysis	101
138	6.2	Safety standards	102
140	6.2.1	DO-178C	102
141	6.2.2	ISO 26262	104
141	6.2.3	Beyond current safety standards and requirements	105
143	6.3	From requirements to verification	105

6.3.1	Formal verification algorithms	106
6.3.2	Resources for verification and computational complexity	107
6.3.3	Invariants and safety requirements	109
6.3.4	Progress requirements	110
6.4	Linear temporal logic	111
6.4.1	Background definitions	112
6.4.2	LTL syntax	113
6.4.3	LTL semantics	113
6.5	Computational tree logic	116
6.5.1	Computational tree logic syntax	116
6.5.2	Computational tree logic semantics	116
6.5.3	Expressiveness of linear temporal logic and computational tree logic	117
6.6	Further reading	118
6.6.1	Checking temporal logic models	118
6.6.2	Planning and synthesis with temporal logics	118
6.6.3	Dense time, signal, and stochastic temporal logics	119
6.6.4	Runtime verification and monitoring	120
6.7	Exercises	121
7	Verifying Invariants	123
7.1	Quick introduction to proving invariants	123
7.2	Reasoning with inductive invariants	125
7.2.1	Invariance and composition	127
7.3	Proving timing-based mutual exclusion	127
7.3.1	Example: Fischer's mutual exclusion	127
7.3.2	Analysis of Fischer's mutual exclusion	130
7.4	Proving inductive invariants without solving ordinary differential equations	133
7.4.1	Example: Checking subtangential conditions	134
7.4.2	Barrier certificates	136
7.5	Satisfiability and satisfiability modulo theories	137
7.5.1	Satisfiability	137
7.5.2	Satisfiability modulo theory	138
7.5.3	Modeling for satisfiability and satisfiability modulo theory	140
7.6	Further reading	141
7.6.1	Finding and learning invariants	141
7.7	Exercises	143

8	Abstractions and Compositional Reasoning	145
8.1	Quick introduction to abstractions: Timing abstraction	145
8.2	Abstraction definitions	148
8.3	Proving abstractions: Simulation relations	149
8.4	Bisimulations and time-abstract bisimulations	152
8.4.1	Untiming and bisimulations	153
8.4.2	Example: Simulation and trace inclusion	154
8.4.3	Backward simulations	154
8.5	Hybridization	155
8.6	Substituting with abstractions	157
8.7	Designing a CEGAR-based cyber-physical verification system	158
8.7.1	Space of abstractions	159
8.7.2	Model checker	162
8.7.3	Counterexample validation	162
8.7.4	Refinement strategy	163
8.8	Further reading	163
8.9	Exercises	163
9	Reachability Analysis	165
9.1	Quick introduction to reachability analysis	165
9.2	Finite automata	166
9.2.1	Finite state reachability	166
9.3	Timed automata	168
9.3.1	Syntax for timed automata	168
9.3.2	Example: Timed light switch	170
9.3.3	Clock equivalence relation on states	170
9.3.4	Control state reachability and region automata	173
9.4	Integral timed automata to rectangular hybrid automata	176
9.4.1	Rational timed automata	176
9.4.2	Multirate automata	176
9.4.3	Rectangular hybrid automata	177
9.5	Undecidability of control state reachability for rectangular hybrid automata	178
9.5.1	Two-counter machines	178
9.5.2	Reduction of control state reachability of rectangular hybrid automata to the halting problem of two-counter machines	179
9.5.3	Initialized rectangular hybrid automata	182
9.6	Relaxing the verification problem	183
9.6.1	Bounded reachability analysis	184
9.7	Data structures for reachability analysis	186

142	9.7.1	Rectangles	186
142	9.7.2	Polytopes	189
148	9.7.3	Zonotopes	192
149	9.7.4	Ellipsoids	193
152	9.8	Exercises	194
152	10	Progress Analysis	197
154	10.1	Quick introduction to progress	197
154	10.2	Termination of discrete-time automata	198
157	10.2.1	Termination with well-founded relations	199
158	10.2.2	Example: UpDown counter	200
159	10.2.3	Termination with disjunctive well-founded relations	201
162	10.2.4	Example: UpDown revisited	202
162	10.3	Self-stabilization	202
163	10.3.1	Example: Distributed minimum spanning tree algorithm	203
163	10.3.2	Stabilization of the minimum spanning tree algorithm	205
163	10.4	Convergence and stability of asynchronous systems without metrics	206
165	10.4.1	Convergence for finite state systems	207
165	10.5	Stability proofs for dynamical systems	208
165	10.6	Stability of hybrid automata	210
166	10.6.1	Common Lyapunov functions	210
166	10.6.2	Multiple Lyapunov functions	211
168	10.6.3	Stability under slow switching: Average dwell time	212
168	10.7	Exercises	214
170	11	Data-Driven Verification	217
173	11.1	Quick introduction to data-driven safety verification	218
176	11.1.1	Discrepancy functions	218
176	11.1.2	BasicSimReach algorithm	219
176	11.1.3	Example: Moore-Greitzer jet engine	222
177	11.2	Computing discrepancy	222
178	11.2.1	Linear dynamical systems	223
178	11.2.2	Nonlinear dynamical systems: Optimization approaches	223
178	11.2.3	Nonlinear models: Local discrepancy	224
179	11.3	Hybrid system verification	226
182	11.3.1	C2E2 verification tool	227
183	11.3.2	Example: Reachability analysis for PulseGen Oscillator with C2E2	228
184	11.4	Example: Powertrain control system	228
186	11.5	Verifying cyber-physical systems with incomplete models	229

11.5.1	Hybrid automata with black-box modules	230
11.5.2	Learning discrepancy from simulations	232
11.5.3	DryVR verification tool	235
11.6	Example: Analyzing risk in automatic emergency braking systems	236
11.7	Example: Autonomous spacecraft rendezvous	237
11.8	Further reading	241
11.8.1	Related approaches, software tools, and applications	241
11.8.2	Limitations and open problems	242
11.8.3	Falsification	243
11.8.4	Statistical model checking	243
11.8.5	Verification for machine learning modules	244
11.9	Exercises	244
Appendix A: Linear Algebra and Real Analysis		247
A.1	Sets and functions	247
A.1.1	Vectors and norms	247
A.1.2	Continuity and derivatives	248
A.1.3	Covers and partitions	248
A.2	Linear functions	249
A.3	Eigenvalues and eigenvectors	249
A.3.1	Positive definite matrices	250
A.3.2	Jordan normal form	250
A.3.3	Matrix norms	251
A.3.4	Interval matrices	251
A.4	Grönwall's inequality	252
Appendix B: Computability and Complexity		255
B.1	Computability	255
B.1.1	Turing machines	255
B.1.2	Configurations and computations	256
B.1.3	Language recognition and decidable languages	257
B.2	Complexity	258
B.2.1	Common complexity classes	258
B.3	Reasoning about the optimality of algorithms	260
B.3.1	Reductions	260
B.3.2	Completeness	261
Appendix C: Specification Language Reference		263
C.1	Conventions	263
C.2	Types	264
C.3	Formal arguments	264

230	C.4	Automaton declaration	265
232	C.5	Action declarations	265
232	C.6	Variables	266
236	C.7	Predicates and expressions	267
237	C.8	Transitions	267
241	C.9	Trajectories	268
241	C.10	Urgency	269
242	C.11	Modeling with nondeterminism	270
243	10.2	Termination of discrete-time automata	271
243	10.2.1	Termination with well-founded relations	271
244	10.2.2	Example: UpDown counting	271
244	10.2.3	Termination with disjunctive well-founded relations	291
244	10.2.4	Example: UpDown revisited	291
247	10.3	Self-stabilization	291
247	10.3.1	Example: Distributed minimum spanning trees	291
247	10.3.2	Stabilization of the minimum spanning tree	291
248	10.4	Convergence and stability of asynchronous systems	291
248	10.4.1	Convergence for finite state systems	291
249	10.5	Stability proofs for dynamical systems	291
249	10.6	Stability of hybrid automata	291
250	10.6.1	Common Lyapunov functions	291
250	10.6.2	Multiple Lyapunov functions	291
251	10.6.3	Stability under slow switching	291
251	10.7	Exercises	291
252	11	Data-Driven Verification	291
252	11.1	Quick introduction to data-driven verification	291
252	11.1.1	Discrepancy functions	291
252	11.1.2	BasicSimReach algorithm	291
256	11.1.3	Example: Moore-Greiner	291
257	11.2	Computing discrepancy	291
258	11.2.1	Linear dynamical systems	291
258	11.2.2	Nonlinear dynamical systems	291
260	11.2.3	Nonlinear models	291
260	11.3	Hybrid system verification	291
261	11.3.1	CE2 verification tool	291
263	11.3.2	Example: Reachability	291
	Appendix A: Linear Algebra and Real Analysis		291
	A.1	Sets and functions	291
	A.1.1	Vectors and norms	291
	A.1.2	Continuity and derivatives	291
	A.1.3	Covers and partitions	291
	A.2	Linear functions	291
	A.3	Eigenvalues and eigenvectors	291
	A.3.1	Positive definite matrices	291
	A.3.2	Jordan normal form	291
	A.3.3	Matrix norms	291
	A.3.4	Interval matrices	291
	A.4	Grönwall's inequality	291
	Appendix B: Computability and Complexity		291
	B.1	Computability	291
	B.1.1	Turing machines	291
	B.1.2	Configurations and computations	291
	B.1.3	Language recognition and decidable languages	291
	B.2	Complexity	291
	B.2.1	Common complexity classes	291
	B.2.2	Reasoning about the optimality of algorithms	291
	B.2.3	Reasoning about the optimality of algorithms	291
	B.3	Reductions	291
	B.3.1	Reductions	291
	B.3.2	Completeness	291
	Appendix C: Specification Languages		291
	C.1	Example: Reachability	291