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What do the shapes of Fig. 1 have in common? Perhaps first we should ask what they are. We'll argue that in one way or another, all are fractals, shapes made of pieces each of which looks like the whole shape. As an important organizing principle of geometry this was first postulated by Benoit Mandelbrot, whose articulation and manifestation of this geometry is *The Fractal Geometry of Nature*.



Figure 1: Some natural fractals.

The picture of the top row is a coastline, an appropriate image to start a gallery of fractals because Benoit Mandelbrot's 1967 paper "How Long is the Coast of Britain?" [2] was the first widely-available result to present a fractal aspect of nature, though Mandelbrot had not yet introduced the word "fractal". This paper does contain the term "self-similarity" which means that a coastline, like a coastline, consists of bits that are similar to the whole. Bays and inlets are decorated with smaller bays and inlets, those in turn by still smaller bays and inlets, for several more levels. With Google Maps, zooming in on a coastline, especially of Norway or Wales, shows similar structures over

The picture of the top row is a view of a hurricane from space. The spiral, itself a spiral of clouds, appears to be surrounded by a collection of smaller spirals, some of which reveal their own tales of still smaller spirals. In fact, this image was shown in a summer workshop, the talk about it was about a cup of latte from which someone has spooned out most of the foam. In fact, we use this picture introduced another fractal image, a fractal image, by which we mean it is difficult to distinguish a