

# CONTENTS

INTRODUCTION .....	vii
<b>1 SELECTED READING ON THE FOUNDATIONS OF ALGEBRA, TOPOLOGY, MATHEMATICAL ANALYSIS AND THE THEORY OF DIFFERENTIAL EQUATIONS .....</b>	<b>1</b>
1.1 Some basic concepts and notation .....	1
1.2 Relations on a set .....	2
1.3 Linear, Euclidean and linear normed spaces .....	3
1.4 Metric spaces .....	5
1.5 Topological spaces .....	5
1.6 Matrices .....	9
1.7 Linear mappings .....	10
1.8 Mathematical analysis .....	12
1.9 Differential equations .....	18
<b>2 FOUNDATIONS OF THE THEORY OF DIFFERENTIABLE MANIFOLDS AND DIFFERENTIABLE MAPPINGS .....</b>	<b>22</b>
2.1 $C^r$ -manifolds .....	22
2.2 $C^r$ -mappings .....	26
2.3 Tangent space to a $C^r$ -manifold .....	27
2.4 $C^r$ -submanifolds .....	32
2.5 $C^r$ -manifolds in $R^N$ .....	35
2.6 Immersion and submersion theorems .....	36
2.7 Regular and critical values of mappings .....	39
2.8 Topology on the space of $C^r$ -mappings .....	41
2.9 Jets .....	45
2.10 Transversality .....	46
2.11 Stratification of algebraic and semi-algebraic manifolds .....	55
2.12 Transversality to stratification .....	61
<b>3 VECTOR FIELDS AND DYNAMICAL SYSTEMS .....</b>	<b>63</b>
3.1 Vector fields on differentiable manifolds .....	63



3.2	Limit properties of dynamical systems	75
3.3	Examples of vector fields	84
3.4	Generic properties of parameter-dependent matrices	86
3.5	Linear dynamical systems and some notions from the theory of non-linear dynamical systems	103
3.6	Grobman–Hartman Theorem	128
3.7	Normal forms of differential equations	143
3.8	Poincaré mapping	159
4	INVARIANT MANIFOLDS	172
4.1	Stable and unstable manifolds	172
4.2	Centre manifolds	183
5	GENERIC BIFURCATIONS OF VECTOR FIELDS AND DIFFEOMORPHISMS	203
5.1	Ljapunov-Schmidt Method	203
5.2	Generic bifurcations of 1-parameter systems of vector fields in neighbourhoods of singular points	214
5.3	Generic bifurcations of 1-parameter systems of diffeomorphisms	238
5.4	Generic bifurcations of 1-parameter systems of vector fields in neighbourhoods of periodic trajectories	258
6	COMPLEMENTARY NOTES ON THE CONTEMPORARY THEORY OF DYNAMICAL SYSTEMS	262
6.1	Generic bifurcations of multi-parameter systems of vector fields	262
6.2	Global theory of dynamical systems	270
6.3	Šilnikov bifurcation	273
6.4	Global Hopf bifurcation	276
6.5	Attractors and chaotic sets	279
	REFERENCES	282
	SUBJECT INDEX	289