

TC	1	Introduction	1	
10		Historical Remarks	1	
20	2	Fundamental Group	2.5.2.9	
30	3.1	Computing the Fundamental Group	3.5.2.9	
40	3.2	Homotopy Equivalence and Retracts	3.5.2.9	
50	3.3	The Fundamental Group of Spheres and Tori	233	
60	3.4	The Seifert-van Kampen Theorem	236	
70	3.4.1	Flowers and Surfaces	3.5.2.6	
80	3.4.2	The Seifert-van Kampen Theorem	3.5.2.8	
90	3.5	Covering spaces	3.5.2.8	
100	3.6	Group Actions and Deck Transformations	3.5.2.8	
110	3.7	Applications	3.5.2.8	
120	3.7.1	Order and Dimension	3.5.2.8	
130	3.7.2	Matter Physics	4.1.2.7	
140		3.7.3	Topological Groups	2.5
150		3.7.4	Homology	3.3.2.9
160		3.7.5	π_1 and Open Subsets	3.5.1.2
170		3.7.6	flows on S^1	3.6.1
180		3.7.7	Chains and Boundaries	3.5.2.3
190		3.7.8	Applications	3.5.2.3
200		3.7.9	Fractals and Combinatorial Topology	3.5.2.3
210		3.7.10	Hyperbolic Geometry	3.5.2.3
220		3.7.11	Surfaces	3.5.2.3
230		3.7.12	Groups	3.5.2.3
240		3.7.13	Simplicial Homology	3.5.2.3
250		3.7.14	Groups	3.5.2.3
260		3.7.15	Cycles and Open Subsets	3.5.2.3
270		3.7.16	Groups	3.5.2.3
280		3.7.17	Applications	3.5.2.3
290		3.7.18	Fractals and Combinatorial Topology	3.5.2.3
300		3.7.19	Hyperbolic Geometry	3.5.2.3
310		3.7.20	Surfaces	3.5.2.3
320		3.7.21	Groups	3.5.2.3
330		3.7.22	Simplicial Homology	3.5.2.3
340		3.7.23	Groups	3.5.2.3
350		3.7.24	Cycles and Open Subsets	3.5.2.3
360		3.7.25	Groups	3.5.2.3
370		3.7.26	Applications	3.5.2.3
380		3.7.27	Fractals and Combinatorial Topology	3.5.2.3
390		3.7.28	Hyperbolic Geometry	3.5.2.3
400		3.7.29	Surfaces	3.5.2.3
410		3.7.30	Groups	3.5.2.3
420		3.7.31	Simplicial Homology	3.5.2.3
430		3.7.32	Groups	3.5.2.3
440		3.7.33	Cycles and Open Subsets	3.5.2.3
450		3.7.34	Groups	3.5.2.3
460		3.7.35	Applications	3.5.2.3
470		3.7.36	Fractals and Combinatorial Topology	3.5.2.3
480		3.7.37	Hyperbolic Geometry	3.5.2.3
490		3.7.38	Surfaces	3.5.2.3
500		3.7.39	Groups	3.5.2.3
510		3.7.40	Simplicial Homology	3.5.2.3
520		3.7.41	Groups	3.5.2.3
530		3.7.42	Cycles and Open Subsets	3.5.2.3
540		3.7.43	Groups	3.5.2.3
550		3.7.44	Applications	3.5.2.3
560		3.7.45	Fractals and Combinatorial Topology	3.5.2.3
570		3.7.46	Hyperbolic Geometry	3.5.2.3
580		3.7.47	Surfaces	3.5.2.3
590		3.7.48	Groups	3.5.2.3
600		3.7.49	Simplicial Homology	3.5.2.3
610		3.7.50	Groups	3.5.2.3
620		3.7.51	Cycles and Open Subsets	3.5.2.3
630		3.7.52	Groups	3.5.2.3
640		3.7.53	Applications	3.5.2.3
650		3.7.54	Fractals and Combinatorial Topology	3.5.2.3
660		3.7.55	Hyperbolic Geometry	3.5.2.3
670		3.7.56	Surfaces	3.5.2.3
680		3.7.57	Groups	3.5.2.3
690		3.7.58	Simplicial Homology	3.5.2.3
700		3.7.59	Groups	3.5.2.3
710		3.7.60	Cycles and Open Subsets	3.5.2.3
720		3.7.61	Groups	3.5.2.3
730		3.7.62	Applications	3.5.2.3
740		3.7.63	Fractals and Combinatorial Topology	3.5.2.3
750		3.7.64	Hyperbolic Geometry	3.5.2.3
760		3.7.65	Surfaces	3.5.2.3
770		3.7.66	Groups	3.5.2.3
780		3.7.67	Simplicial Homology	3.5.2.3
790		3.7.68	Groups	3.5.2.3
800		3.7.69	Cycles and Open Subsets	3.5.2.3
810		3.7.70	Groups	3.5.2.3
820		3.7.71	Applications	3.5.2.3
830		3.7.72	Fractals and Combinatorial Topology	3.5.2.3
840		3.7.73	Hyperbolic Geometry	3.5.2.3
850		3.7.74	Surfaces	3.5.2.3
860		3.7.75	Groups	3.5.2.3
870		3.7.76	Simplicial Homology	3.5.2.3
880		3.7.77	Groups	3.5.2.3
890		3.7.78	Cycles and Open Subsets	3.5.2.3
900		3.7.79	Groups	3.5.2.3
910		3.7.80	Applications	3.5.2.3
920		3.7.81	Fractals and Combinatorial Topology	3.5.2.3
930		3.7.82	Hyperbolic Geometry	3.5.2.3
940		3.7.83	Surfaces	3.5.2.3
950		3.7.84	Groups	3.5.2.3
960		3.7.85	Simplicial Homology	3.5.2.3
970		3.7.86	Groups	3.5.2.3
980		3.7.87	Cycles and Open Subsets	3.5.2.3
990		3.7.88	Groups	3.5.2.3
1000		3.7.89	Applications	3.5.2.3
1010		3.7.90	Fractals and Combinatorial Topology	3.5.2.3
1020		3.7.91	Hyperbolic Geometry	3.5.2.3
1030		3.7.92	Surfaces	3.5.2.3
1040		3.7.93	Groups	3.5.2.3
1050		3.7.94	Simplicial Homology	3.5.2.3
1060		3.7.95	Groups	3.5.2.3
1070		3.7.96	Cycles and Open Subsets	3.5.2.3
1080		3.7.97	Groups	3.5.2.3
1090		3.7.98	Applications	3.5.2.3
1100		3.7.99	Fractals and Combinatorial Topology	3.5.2.3
1110		3.7.100	Hyperbolic Geometry	3.5.2.3
1120		3.7.101	Surfaces	3.5.2.3
1130		3.7.102	Groups	3.5.2.3
1140		3.7.103	Simplicial Homology	3.5.2.3
1150		3.7.104	Groups	3.5.2.3
1160		3.7.105	Cycles and Open Subsets	3.5.2.3
1170		3.7.106	Groups	3.5.2.3
1180		3.7.107	Applications	3.5.2.3
1190		3.7.108	Fractals and Combinatorial Topology	3.5.2.3
1200		3.7.109	Hyperbolic Geometry	3.5.2.3
1210		3.7.110	Surfaces	3.5.2.3
1220		3.7.111	Groups	3.5.2.3
1230		3.7.112	Simplicial Homology	3.5.2.3
1240		3.7.113	Groups	3.5.2.3
1250		3.7.114	Cycles and Open Subsets	3.5.2.3
1260		3.7.115	Groups	3.5.2.3
1270		3.7.116	Applications	3.5.2.3
1280		3.7.117	Fractals and Combinatorial Topology	3.5.2.3
1290		3.7.118	Hyperbolic Geometry	3.5.2.3
1300		3.7.119	Surfaces	3.5.2.3
1310		3.7.120	Groups	3.5.2.3
1320		3.7.121	Simplicial Homology	3.5.2.3
1330		3.7.122	Groups	3.5.2.3
1340		3.7.123	Cycles and Open Subsets	3.5.2.3
1350		3.7.124	Groups	3.5.2.3
1360		3.7.125	Applications	3.5.2.3
1370		3.7.126	Fractals and Combinatorial Topology	3.5.2.3
1380		3.7.127	Hyperbolic Geometry	3.5.2.3
1390		3.7.128	Surfaces	3.5.2.3
1400		3.7.129	Groups	3.5.2.3
1410		3.7.130	Simplicial Homology	3.5.2.3
1420		3.7.131	Groups	3.5.2.3
1430		3.7.132	Cycles and Open Subsets	3.5.2.3
1440		3.7.133	Groups	3.5.2.3
1450		3.7.134	Applications	3.5.2.3
1460		3.7.135	Fractals and Combinatorial Topology	3.5.2.3
1470		3.7.136	Hyperbolic Geometry	3.5.2.3
1480		3.7.137		

2.3	<i>Identification Spaces and Compactness</i>	57
2.4	<i>Connectedness and path-connectedness</i>	61
2.5	<i>Cantor Sets</i>	67
2.6	<i>Application: Compact Sets in Population Dynamics and Fractals</i>	71
3	<i>Manifolds and Complexes</i>	79
3.1	<i>Manifolds</i>	79
3.2	<i>Triangulations</i>	90
3.3	<i>Classification of Surfaces</i>	97
3.3.1	<i>Gluing Disks</i>	98
3.3.2	<i>Planar Models</i>	99
3.3.3	<i>Classification of Surfaces</i>	103
3.4	<i>Euler Characteristic</i>	110
3.5	<i>Topological Groups</i>	114
3.6	<i>Group Actions and Orbit Spaces</i>	126
3.6.1	<i>Flows on Tori</i>	131
3.7	<i>Applications</i>	136
3.7.1	<i>Robotic Coordination and Configuration Spaces</i>	136
3.7.2	<i>Geometry of Manifolds</i>	141
3.7.3	<i>The Topology of the Universe</i>	146
4	<i>Homotopy and the Winding Number</i>	159
4.1	<i>Homotopy and Paths</i>	160
4.2	<i>The Winding Number</i>	164
4.3	<i>Degrees of Maps</i>	174
4.4	<i>The Brouwer Fixed Point Theorem</i>	176
4.5	<i>The Borsuk–Ulam Theorem</i>	179
4.6	<i>Vector Fields and the Poincaré Index Theorem</i>	180
4.7	<i>Applications I</i>	187
4.7.1	<i>The Fundamental Theorem of Algebra</i>	187
4.7.2	<i>Sandwiches</i>	187
4.7.3	<i>Game Theory and Nash Equilibria</i>	190
4.8	<i>Applications II: Calculus</i>	194
4.8.1	<i>Vector Fields, Path Integrals, and the Winding Number</i>	194
4.8.2	<i>Vector Fields on Surfaces</i>	201
4.8.3	<i>Index Theory for n-Symmetry Fields</i>	213

4.9	<i>Index Theory in Computer Graphics</i>	214
5	<i>Fundamental Group</i>	219
5.1	<i>Definition and Basic Properties</i>	219
5.2	<i>Homotopy Equivalence and Retracts</i>	226
5.3	<i>The Fundamental Group of Spheres and Tori</i>	233
5.4	<i>The Seifert–van Kampen Theorem</i>	236
5.4.1	<i>Flowers and Surfaces</i>	236
5.4.2	<i>The Seifert–van Kampen Theorem</i>	238
5.5	<i>Covering spaces</i>	244
5.6	<i>Group Actions and Deck Transformations</i>	252
5.7	<i>Applications</i>	257
5.7.1	<i>Order and Emergent Patterns in Condensed Matter Physics</i>	257
6	<i>Homology</i>	269
6.1	<i>Δ-complexes</i>	270
6.2	<i>Chains and Boundaries</i>	273
6.3	<i>Examples and Computations</i>	279
6.4	<i>Singular Homology</i>	285
6.5	<i>Homotopy Invariance</i>	288
6.6	<i>Brouwer Fixed Point Theorem for D^n</i>	296
6.7	<i>Homology and the Fundamental Group</i>	297
6.8	<i>Betti Numbers and the Euler Characteristic</i>	300
6.9	<i>Computational Homology</i>	301
6.9.1	<i>Computing Betti Numbers</i>	302
6.9.2	<i>Building a Filtration</i>	304
6.9.3	<i>Persistent Homology</i>	307
	<i>Appendix A Knot Theory</i>	313
	<i>Appendix B Groups</i>	321
	<i>Appendix C Perspectives in Topology</i>	325
C.1	<i>Point Set Topology</i>	325
C.2	<i>Geometric Topology</i>	326
C.3	<i>Algebraic Topology</i>	327
C.4	<i>Combinatorial Topology</i>	329
C.5	<i>Differential Topology</i>	331

References	333
4. Connectedness and path-connectedness	333
Bibliography	333
BAS.1 Application: Combinatorial Mathematics	333
Index	337
2.2.1 Two-manifolds and higher-dimensional analogues	337
2.2.2 Manifolds and Complementarity	337
2.2.3.1 Manifolds	337
2.2.3.2 Triangulations	337
2.2.3.3 Classification of Surfaces	337
2.2.4.1 Curves and Degree	337
2.2.5 Planar Models	337
2.3 Other Applications	337
2.3.1 Combinatorial Games	337
2.3.2.1 Submanifolds and Orbit Spaces	337
2.3.2.2 Winding Number	337
2.3.3.1 Homotopy	337
2.3.3.2 Curvature	337
2.3.3.3 Geometry of Manifolds	337
2.3.3.4 The Riemannian Metric	337
2.3.4.1 Homology and the Fundamental Group	337
2.3.4.2 Homotopy and Paths	337
2.3.4.3 The Winding Number	337
2.3.4.4 Degrees of Maps	337
2.3.4.5 The Brouwer Fixed Point Theorem	337
2.3.4.6 Vector Fields and the Poincaré Index Theorem	337
2.3.4.7 Applications I	337
2.3.5.1 The Fundamental Theorem of Algebra	337
2.3.5.2 Sandwiches	337
2.3.5.3 Game Theory	337
2.3.6.1 Applications II: Calculus	337
2.3.6.2 Vector Fields, Path Integrals	337
2.3.6.3 Number Theory	337
2.3.6.4 Vector Fields on Surfaces	337
2.3.6.5 Index Theory for Differential Equations	337