

Contents

Preface	xxv
Introduction	xxix
1.1 Preliminaries	xxxii
1.2 Cardinality	xxxv
1 Continuity	1
1.1 Continuity and Open Sets in \mathbb{R}^n	1
1.2 Continuity and Open Sets in Topological Spaces	6
1.3 Metric, Product, and Quotient Topologies	9
1.4 Subsets of Topological Spaces	19
1.5 Continuous Functions and Topological Equivalence	27
1.6 Surfaces	34
1.7 Application: Chaos in Dynamical Systems	39
1.7.1 History of Chaos	39
1.7.2 A Simple Example	40
1.7.3 Notions of Chaos	41
2 Compactness and Connectedness	47
2.1 Closed Bounded Subsets of \mathbb{R}	47
2.2 Compact Spaces	51

2.3	<i>Identification Spaces and Compactness</i>	57
2.4	<i>Connectedness and path-connectedness</i>	61
2.5	<i>Cantor Sets</i>	67
2.6	<i>Application: Compact Sets in Population Dynamics and Fractals</i>	71
3	<i>Manifolds and Complexes</i>	79
3.1	<i>Manifolds</i>	79
3.2	<i>Triangulations</i>	90
3.3	<i>Classification of Surfaces</i>	97
3.3.1	<i>Gluing Disks</i>	98
3.3.2	<i>Planar Models</i>	99
3.3.3	<i>Classification of Surfaces</i>	103
3.4	<i>Euler Characteristic</i>	110
3.5	<i>Topological Groups</i>	114
3.6	<i>Group Actions and Orbit Spaces</i>	126
3.6.1	<i>Flows on Tori</i>	131
3.7	<i>Applications</i>	136
3.7.1	<i>Robotic Coordination and Configuration Spaces</i>	136
3.7.2	<i>Geometry of Manifolds</i>	141
3.7.3	<i>The Topology of the Universe</i>	146
4	<i>Homotopy and the Winding Number</i>	159
4.1	<i>Homotopy and Paths</i>	160
4.2	<i>The Winding Number</i>	164
4.3	<i>Degrees of Maps</i>	174
4.4	<i>The Brouwer Fixed Point Theorem</i>	176
4.5	<i>The Borsuk–Ulam Theorem</i>	179
4.6	<i>Vector Fields and the Poincaré Index Theorem</i>	180
4.7	<i>Applications I</i>	187
4.7.1	<i>The Fundamental Theorem of Algebra</i>	187
4.7.2	<i>Sandwiches</i>	187
4.7.3	<i>Game Theory and Nash Equilibria</i>	190
4.8	<i>Applications II: Calculus</i>	194
4.8.1	<i>Vector Fields, Path Integrals, and the Winding Number</i>	194
4.8.2	<i>Vector Fields on Surfaces</i>	201
4.8.3	<i>Index Theory for n-Symmetry Fields</i>	213

4.9	<i>Index Theory in Computer Graphics</i>	214
5	<i>Fundamental Group</i>	219
5.1	<i>Definition and Basic Properties</i>	219
5.2	<i>Homotopy Equivalence and Retracts</i>	226
5.3	<i>The Fundamental Group of Spheres and Tori</i>	233
5.4	<i>The Seifert–van Kampen Theorem</i>	236
5.4.1	<i>Flowers and Surfaces</i>	236
5.4.2	<i>The Seifert–van Kampen Theorem</i>	238
5.5	<i>Covering spaces</i>	244
5.6	<i>Group Actions and Deck Transformations</i>	252
5.7	<i>Applications</i>	257
5.7.1	<i>Order and Emergent Patterns in Condensed Matter Physics</i>	257
6	<i>Homology</i>	269
6.1	<i>Δ-complexes</i>	270
6.2	<i>Chains and Boundaries</i>	273
6.3	<i>Examples and Computations</i>	279
6.4	<i>Singular Homology</i>	285
6.5	<i>Homotopy Invariance</i>	288
6.6	<i>Brouwer Fixed Point Theorem for D^n</i>	296
6.7	<i>Homology and the Fundamental Group</i>	297
6.8	<i>Betti Numbers and the Euler Characteristic</i>	300
6.9	<i>Computational Homology</i>	301
6.9.1	<i>Computing Betti Numbers</i>	302
6.9.2	<i>Building a Filtration</i>	304
6.9.3	<i>Persistent Homology</i>	307
	<i>Appendix A Knot Theory</i>	313
	<i>Appendix B Groups</i>	321
	<i>Appendix C Perspectives in Topology</i>	325
C.1	<i>Point Set Topology</i>	325
C.2	<i>Geometric Topology</i>	326
C.3	<i>Algebraic Topology</i>	327
C.4	<i>Combinatorial Topology</i>	329
C.5	<i>Differential Topology</i>	331

References		333
2.4 Connectedness and path-connectedness		76
Bibliography		333
Index		337
2.1 The Fundamental Group		337
2.2 Homotopy Equivalence and Retractions		337
2.3 The Fundamental Group of Spheres and Tori		337
2.4 The Seifert-van Kampen Theorem		337
2.4.1 Flowers and Surfaces		337
2.4.2 The Seifert-van Kampen Theorem		337
2.4.3 Classification of Surfaces		337
2.5 Covering Spaces		337
2.6 Group Actions and Deck Transformations		337
2.7 Applications		337
2.7.1 Order and Singularity		337
2.7.2 Planar Models		337
2.7.3 Order and Singularity		337
2.7.4 Planar Models		337
2.7.5 Order and Singularity		337
2.7.6 Planar Models		337
2.7.7 Order and Singularity		337
2.7.8 Planar Models		337
2.7.9 Order and Singularity		337
2.7.10 Planar Models		337
2.7.11 Order and Singularity		337
2.7.12 Planar Models		337
2.7.13 Order and Singularity		337
2.7.14 Planar Models		337
2.7.15 Order and Singularity		337
2.7.16 Planar Models		337
2.7.17 Order and Singularity		337
2.7.18 Planar Models		337
2.7.19 Order and Singularity		337
2.7.20 Planar Models		337
2.7.21 Order and Singularity		337
2.7.22 Planar Models		337
2.7.23 Order and Singularity		337
2.7.24 Planar Models		337
2.7.25 Order and Singularity		337
2.7.26 Planar Models		337
2.7.27 Order and Singularity		337
2.7.28 Planar Models		337
2.7.29 Order and Singularity		337
2.7.30 Planar Models		337
2.7.31 Order and Singularity		337
2.7.32 Planar Models		337
2.7.33 Order and Singularity		337
2.7.34 Planar Models		337
2.7.35 Order and Singularity		337
2.7.36 Planar Models		337
2.7.37 Order and Singularity		337
2.7.38 Planar Models		337
2.7.39 Order and Singularity		337
2.7.40 Planar Models		337
2.7.41 Order and Singularity		337
2.7.42 Planar Models		337
2.7.43 Order and Singularity		337
2.7.44 Planar Models		337
2.7.45 Order and Singularity		337
2.7.46 Planar Models		337
2.7.47 Order and Singularity		337
2.7.48 Planar Models		337
2.7.49 Order and Singularity		337
2.7.50 Planar Models		337
2.7.51 Order and Singularity		337
2.7.52 Planar Models		337
2.7.53 Order and Singularity		337
2.7.54 Planar Models		337
2.7.55 Order and Singularity		337
2.7.56 Planar Models		337
2.7.57 Order and Singularity		337
2.7.58 Planar Models		337
2.7.59 Order and Singularity		337
2.7.60 Planar Models		337
2.7.61 Order and Singularity		337
2.7.62 Planar Models		337
2.7.63 Order and Singularity		337
2.7.64 Planar Models		337
2.7.65 Order and Singularity		337
2.7.66 Planar Models		337
2.7.67 Order and Singularity		337
2.7.68 Planar Models		337
2.7.69 Order and Singularity		337
2.7.70 Planar Models		337
2.7.71 Order and Singularity		337
2.7.72 Planar Models		337
2.7.73 Order and Singularity		337
2.7.74 Planar Models		337
2.7.75 Order and Singularity		337
2.7.76 Planar Models		337
2.7.77 Order and Singularity		337
2.7.78 Planar Models		337
2.7.79 Order and Singularity		337
2.7.80 Planar Models		337
2.7.81 Order and Singularity		337
2.7.82 Planar Models		337
2.7.83 Order and Singularity		337
2.7.84 Planar Models		337
2.7.85 Order and Singularity		337
2.7.86 Planar Models		337
2.7.87 Order and Singularity		337
2.7.88 Planar Models		337
2.7.89 Order and Singularity		337
2.7.90 Planar Models		337
2.7.91 Order and Singularity		337
2.7.92 Planar Models		337
2.7.93 Order and Singularity		337
2.7.94 Planar Models		337
2.7.95 Order and Singularity		337
2.7.96 Planar Models		337
2.7.97 Order and Singularity		337
2.7.98 Planar Models		337
2.7.99 Order and Singularity		337
2.7.100 Planar Models		337