

# CONTENTS

## Preface to the second edition

vii

## Preface

ix

## 1 Categories

- 1.1 Introduction 1
- 1.2 Functions of sets 3
- 1.3 Definition of a category 4
- 1.4 Examples of categories 5
- 1.5 Isomorphisms 12
- 1.6 Constructions on categories 14
- 1.7 Free categories 18
- 1.8 Foundations: large, small, and locally small 23
- 1.9 Exercises 25

## 2 Abstract structures

- 2.1 Epis and monos 29
- 2.2 Initial and terminal objects 33
- 2.3 Generalized elements 35
- 2.4 Products 38
- 2.5 Examples of products 41
- 2.6 Categories with products 46
- 2.7 Hom-sets 48
- 2.8 Exercises 50

## 3 Duality

- 3.1 The duality principle 53
- 3.2 Coproducts 55
- 3.3 Equalizers 62
- 3.4 Coequalizers 65
- 3.5 Exercises 71

## 4 Groups and categories

- 4.1 Groups in a category 75
- 4.2 The category of groups 80
- 4.3 Groups as categories 83
- 4.4 Finitely presented categories 85
- 4.5 Exercises 87



<b>5</b>	<b>Limits and colimits</b>	89
5.1	Subobjects	89
5.2	Pullbacks	91
5.3	Properties of pullbacks	95
5.4	Limits	100
5.5	Preservation of limits	105
5.6	Colimits	108
5.7	Exercises	114
<b>6</b>	<b>Exponentials</b>	119
6.1	Exponential in a category	119
6.2	Cartesian closed categories	122
6.3	Heyting algebras	129
6.4	Propositional calculus	131
6.5	Equational definition of CCC	134
6.6	$\lambda$ -calculus	135
6.7	Variable sets	140
6.8	Exercises	144
<b>7</b>	<b>Naturality</b>	147
7.1	Category of categories	147
7.2	Representable structure	149
7.3	Stone duality	153
7.4	Naturality	155
7.5	Examples of natural transformations	157
7.6	Exponentials of categories	161
7.7	Functor categories	164
7.8	Monoidal categories	168
7.9	Equivalence of categories	171
7.10	Examples of equivalence	175
7.11	Exercises	181
<b>8</b>	<b>Categories of diagrams</b>	185
8.1	Set-valued functor categories	185
8.2	The Yoneda embedding	187
8.3	The Yoneda lemma	188
8.4	Applications of the Yoneda lemma	193
8.5	Limits in categories of diagrams	194
8.6	Colimits in categories of diagrams	195
8.7	Exponentials in categories of diagrams	199
8.8	Topoi	201
8.9	Exercises	203
<b>9</b>	<b>Adjoints</b>	207
9.1	Preliminary definition	207
9.2	Hom-set definition	211



9.3	Examples of adjoints	215
9.4	Order adjoints	219
9.5	Quantifiers as adjoints	221
9.6	RAPL	225
9.7	Locally cartesian closed categories	231
9.8	Adjoint functor theorem	239
9.9	Exercises	248
<b>10</b>	<b>Monads and algebras</b>	<b>253</b>
10.1	The triangle identities	253
10.2	Monads and adjoints	255
10.3	Algebras for a monad	259
10.4	Comonads and coalgebras	264
10.5	Algebras for endofunctors	266
10.6	Exercises	274
	<b>Solutions to selected exercises</b>	<b>279</b>
	<b>References</b>	<b>303</b>
	<b>Index</b>	<b>305</b>



We think of the composition  $g \circ f$  as a sort of "product" of the functions  $f$  and  $g$ , and consider abstract "algebras" of the sort arising from collections of functions. A category is just such an "algebra," consisting of objects  $A, B, C, \dots$  and arrows  $f: A \rightarrow B, g: B \rightarrow C, \dots$ , that are closed under composition and satisfy certain conditions typical of the composition of functions. A precise definition is given later in this chapter.

A branch of abstract algebra, category theory was invented in the tradition of Felix Klein's *Erlanger Programm*, as a way of studying and characterizing different kinds of mathematical structures in terms of their "admissible transformations." The general notion of a category provides a characterization of the notion of a "structure-preserving transformation," and thereby of a species of structures admitting such transformations.

The historical development of the subject has been, very roughly, as follows:

1945: Eilenberg and Mac Lane's "General theory of natural equivalences" was the original paper, in which the theory was first formulated.

Late 1940s: The main applications were originally in the fields of algebraic topology, particularly homology theory, and abstract algebra.