

# Contents

Preface	1
Rough guide to notation	3
INTRODUCTION	5
<b>1. FORMALISM <math>\leftrightarrow</math> 2. SIMPLE SYSTEMS</b>	<b>10</b>
<b>1.1 Space of quantum states</b> . . . . .	10
Hilbert space. Rigged Hilbert space . . . . .	10
Dirac notation . . . . .	12
Sum & product of spaces . . . . .	14
<b>2.1 Examples of quantum Hilbert spaces</b> . . . . .	15
Single structureless particle with spin 0 or $\frac{1}{2}$ . . . . .	15
2 distinguishable/indistinguishable particles. Bosons & fermions . . . . .	17
Ensembles of $N \geq 2$ particles . . . . .	19
<b>1.2 Representation of observables</b> . . . . .	21
Observables as Hermitian operators. Basic properties . . . . .	21
Eigenvalues & eigenvectors in finite & infinite dimension . . . . .	23
Discrete & continuous spectrum. Spectral decomposition . . . . .	25
<b>2.2 Examples of quantum operators</b> . . . . .	27
Spin- $\frac{1}{2}$ operators . . . . .	27
Coordinate & momentum . . . . .	29
Hamiltonian of free particle & particle in potential . . . . .	30
Orbital angular momentum. Isotropic Hamiltonians . . . . .	33
Hamiltonian of a particle in electromagnetic field . . . . .	37
<b>1.3 Compatible and incompatible observables</b> . . . . .	39
Compatible observables. Complete set . . . . .	39
Incompatible observables. Uncertainty relation . . . . .	41
Analogy with Poisson brackets . . . . .	42
Equivalent representations . . . . .	43
<b>2.3 Examples of commuting &amp; noncommuting operators</b> . . . . .	44
Coordinate, momentum & associated representations . . . . .	44
Angular momentum components . . . . .	47
Complete sets of commuting operators for structureless particle . . . . .	49
<b>1.4 Representation of physical transformations</b> . . . . .	50
Properties of unitary operators . . . . .	50
Canonical & symmetry transformations . . . . .	52
Basics of group theory . . . . .	54



<b>2.4 Fundamental spatio-temporal symmetries</b> . . . . .	56
Space translation . . . . .	57
Space rotation . . . . .	58
Space inversion . . . . .	61
Time translation & reversal. Galilean transformations . . . . .	62
Symmetry & degeneracy . . . . .	64
<b>1.5 Unitary evolution of quantum systems</b> . . . . .	65
Nonstationary Schrödinger equation. Flow. Continuity equation. . . . .	65
Conservation laws & symmetries . . . . .	67
Energy $\times$ time uncertainty. (Non)exponential decay . . . . .	68
Hamiltonians depending on time. Dyson series . . . . .	71
Schrödinger, Heisenberg & Dirac description . . . . .	73
Green operator. Single-particle propagator . . . . .	74
<b>2.5 Examples of quantum evolution</b> . . . . .	76
Two-level system . . . . .	76
Free particle . . . . .	77
Coherent states in harmonic oscillator . . . . .	79
Spin in rotating magnetic field . . . . .	81
<b>1.6 Quantum measurement</b> . . . . .	83
State vector reduction & consequences . . . . .	83
EPR situation. Interpretation problems . . . . .	85
<b>2.6 Implications &amp; applications of quantum measurement</b> . . . . .	89
Paradoxes of quantum measurement . . . . .	89
Applications of quantum measurement . . . . .	91
Hidden variables. Bell inequalities. Nonlocality . . . . .	92
<b>1.7 Quantum statistical physics</b> . . . . .	94
Pure and mixed states. Density operator . . . . .	95
Entropy. Canonical ensemble . . . . .	96
Wigner distribution function . . . . .	98
Density operator for open systems . . . . .	99
Evolution of density operator: closed & open systems . . . . .	101
<b>2.7 Examples of statistical description</b> . . . . .	104
Harmonic oscillator at nonzero temperature . . . . .	104
Coherent superposition vs. statistical mixture . . . . .	105
Density operator and decoherence for a two-state system . . . . .	106
<b>3. QUANTUM-CLASSICAL CORRESPONDENCE</b> . . . . .	108
<b>3.1 Classical limit of quantum mechanics</b> . . . . .	108
The limit $\hbar \rightarrow 0$ . . . . .	108
Ehrenfest theorem. Role of decoherence . . . . .	109



<b>3.2 WKB approximation</b>	112
Classical Hamilton-Jacobi theory	112
WKB equations & interpretation	114
Quasiclassical approximation	115
<b>3.3 Feynman integral</b>	118
Formulation of quantum mechanics in terms of trajectories	118
Application to the Aharonov-Bohm effect	119
Application to the density of states	120
<b>4. ANGULAR MOMENTUM</b>	<b>123</b>
<b>4.1 General features of angular momentum</b>	123
Eigenvalues and ladder operators	123
Addition of two angular momenta	125
Addition of three angular momenta	128
<b>4.2 Irreducible tensor operators</b>	129
Euler angles. Wigner functions. Rotation group irreps	129
Spherical tensors. Wigner-Eckart theorem	130
<b>5. APPROXIMATION TECHNIQUES</b>	<b>133</b>
<b>5.1 Variational method</b>	133
Dynamical & stationary variational principle. Ritz method	133
<b>5.2 Stationary perturbation method</b>	136
General setup & equations	136
Nondegenerate case	138
Degenerate case	139
Application in atomic physics	141
Application to level dynamics	145
Driven systems. Adiabatic approximation	147
<b>5.3 Nonstationary perturbation method</b>	149
General formalism	149
Step perturbation	152
Exponential & periodic perturbations	154
Application to stimulated electromagnetic transitions	155
<b>6. SCATTERING THEORY</b>	<b>157</b>
<b>6.1 Elementary description of elastic scattering</b>	158
Scattering by fixed potential. Cross section	158
Two-body problem. Center-of-mass system	159
Effect of particle indistinguishability in cross section	160
<b>6.2 Perturbative approach the scattering problem</b>	161

Lippmann-Schwinger equation . . . . .	161
Born series for scattering amplitude . . . . .	164
<b>6.3 Method of partial waves . . . . .</b>	<b>166</b>
Expression of elastic scattering in terms of spherical waves . . . . .	166
Inclusion of inelastic scattering . . . . .	172
Low-energy & resonance scattering . . . . .	174
<b>7. MANY-BODY SYSTEMS . . . . .</b>	<b>175</b>
<b>7.1 Formalism of particle creation/annihilation operators . . . . .</b>	<b>176</b>
Hilbert space of bosons & fermions . . . . .	176
Bosonic & fermionic creation/annihilation operators . . . . .	177
Operators in bosonic & fermionic $N$ -particle spaces . . . . .	181
Quantization of electromagnetic field . . . . .	186
<b>7.2 Many-body techniques . . . . .</b>	<b>189</b>
Fermionic mean field & Hartree-Fock method . . . . .	189
Bosonic condensates & Hartree-Bose method . . . . .	192
Pairing & BCS method . . . . .	193
Quantum gases . . . . .	198

