Contents

List of Contributors *xiii*
Foreword *xvi*

1 Introduction to Geostatistical Functional Data Analysis *1*
Jorge Mateu and Ramón Giraldo
1.1 Spatial Statistics *1*
1.2 Spatial Geostatistics *7*
1.2.1 Regionalized Variables *7*
1.2.2 Random Functions *7*
1.2.3 Stationarity and Intrinsic Hypothesis *9*
1.3 Spatiotemporal Geostatistics *12*
1.3.1 Relevant Spatiotemporal Concepts *12*
1.3.2 Spatiotemporal Kriging *16*
1.3.3 Spatiotemporal Covariance Models *17*
1.4 Functional Data Analysis in Brief *18*
References *22*

Part I Mathematical and Statistical Foundations *27*

2 Mathematical Foundations of Functional Kriging in Hilbert Spaces and Riemannian Manifolds *29*
Alessandra Menafoglio, Davide Pigoli, and Piercesare Secchi
2.1 Introduction *29*
2.2 Definitions and Assumptions *30*
2.3 Kriging Prediction in Hilbert Space: A Trace Approach *33*
2.3.1 Ordinary and Universal Kriging in Hilbert Spaces *33*
2.3.2 Estimating the Drift *36*
2.3.3 An Example: Trace-Variogram in Sobolev Spaces *37*

2.3.4 An Application to Nonstationary Prediction of Temperatures Profiles *39*
2.4 An Operatorial Viewpoint to Kriging *42*
2.5 Kriging for Manifold-Valued Random Fields *45*
2.5.1 Residual Kriging *45*
2.5.2 An Application to Positive Definite Matrices *47*
2.5.3 Validity of the Local Tangent Space Approximation *49*
2.6 Conclusion and Further Research *53*
References *53*

3 Universal, Residual, and External Drift Functional Kriging *55*
Maria Franco-Villoria and Rosaria Ignaccolo
3.1 Introduction *56*
3.2 Universal Kriging for Functional Data (UKFD) *56*
3.3 Residual Kriging for Functional Data (ResKFD) *58*
3.4 Functional Kriging with External Drift (FKED) *60*
3.5 Accounting for Spatial Dependence in Drift Estimation *61*
3.5.1 Drift Selection *62*
3.6 Uncertainty Evaluation *62*
3.7 Implementation Details in R *64*
3.7.1 *Example*: Air Pollution Data *64*
3.8 Conclusions *69*
References *71*

4 Extending Functional Kriging When Data Are Multivariate Curves: Some Technical Considerations and Operational Solutions *73*
David Nerini, Claude Manté, and Pascal Monestiez
4.1 Introduction *73*
4.2 Principal Component Analysis for Curves *74*
4.2.1 Karhunen–Loève Decomposition *74*
4.2.2 Dealing with a Sample *76*
4.3 Functional Kriging in a Nutshell *78*
4.3.1 Solution Based on Basis Functions *79*
4.3.2 Estimation of Spatial Covariances *81*
4.4 An Example with the Precipitation Observations *82*
4.4.1 Fitting Variogram Model *83*
4.4.2 Making Prediction *83*
4.5 Functional Principal Component Kriging *85*
4.6 Multivariate Kriging with Functional Data *88*

4.6.1 Multivariate FPCA *91*
4.6.2 MFPCA Displays *93*
4.6.3 Multivariate Functional Principal Component Kriging *94*
4.6.4 Mixing Temperature and Precipitation Curves *96*
4.7 Discussion *98*
4.A Appendices *100*
4.A.1 Computation of the Kriging Variance *100*
References *102*

5 Geostatistical Analysis in Bayes Spaces: Probability Densities and Compositional Data *104*
Alessandra Menafoglio, Piercesare Secchi, and Alberto Guadagnini
5.1 Introduction and Motivations *104*
5.2 Bayes Hilbert Spaces: Natural Spaces for Functional Compositions *105*
5.3 A Motivating Case Study: Particle-Size Data in Heterogeneous Aquifers – Data Description *108*
5.4 Kriging Stationary Functional Compositions *110*
5.4.1 Model Description *110*
5.4.2 Data Preprocessing *112*
5.4.3 An Example of Application *113*
5.4.4 Uncertainty Assessment *116*
5.5 Analyzing Nonstationary Fields of FCs *119*
5.6 Conclusions and Perspectives *123*
References *124*

6 Spatial Functional Data Analysis for Probability Density Functions: Compositional Functional Data vs. Distributional Data Approach *128*
Elvira Romano, Antonio Irpino, and Jorge Mateu
6.1 FDA and SDA When Data Are Densities *130*
6.1.1 Features of Density Functions as Compositional Functional Data *131*
6.1.2 Features of Density Functions as Distributional Data *135*
6.2 Measures of Spatial Association for Georeferenced Density Functions *138*
6.2.1 Identification of Spatial Clusters by Spatial Association Measures for Density Functions *139*
6.3 Real Data Analysis *141*
6.3.1 The SDA Distributional Approach *143*
6.3.2 The Compositional–Functional Approach *145*
6.3.3 Discussion *147*

6.4 Conclusion *149*
Acknowledgments *151*
References *151*

Part II Statistical Techniques for Spatially Correlated Functional Data *155*

7 Clustering Spatial Functional Data *157*
Vincent Vandewalle, Cristian Preda, and Sophie Dabo-Niang
7.1 Introduction *157*
7.2 Model-Based Clustering for Spatial Functional Data *158*
7.2.1 The Expectation–Maximization (EM) Algorithm *160*
7.2.1.1 E Step *161*
7.2.1.2 M Step *161*
7.2.2 Model Selection *161*
7.3 Descendant Hierarchical Classification (HC) Based on Centrality Methods *162*
7.3.1 Methodology *164*
7.4 Application *165*
7.4.1 Model-Based Clustering *167*
7.4.2 Hierarchical Classification *169*
7.5 Conclusion *171*
References *172*

8 Nonparametric Statistical Analysis of Spatially Distributed Functional Data *175*
Sophie Dabo-Niang, Camille Ternynck, Baba Thiam, and Anne-Françoise Yao
8.1 Introduction *175*
8.2 Large Sample Properties *178*
8.2.1 Uniform Almost Complete Convergence *180*
8.3 Prediction *181*
8.4 Numerical Results *184*
8.4.1 Bandwidth Selection Procedure *184*
8.4.2 Simulation Study *185*
8.5 Conclusion *193*
8.A Appendix *194*
8.A.1 Some Preliminary Results for the Proofs *194*
8.A.2 Proofs *196*
8.A.2.1 Proof of Theorem 8.1 *196*
8.A.2.2 Proof of Lemma A.3 *196*

8.A.2.3 Proof of Lemma A.4 *196*
8.A.2.4 Proof of Lemma A.5 *201*
8.A.2.5 Proof of Lemma A.6 *201*
8.A.2.6 Proof of Theorem 8.2 *202*
References *207*

9 A Nonparametric Algorithm for Spatially Dependent Functional Data: Bagging Voronoi for Clustering, Dimensional Reduction, and Regression *211*
Valeria Vitelli, Federica Passamonti, Simone Vantini, and Piercesare Secchi
9.1 Introduction *211*
9.2 The Motivating Application *212*
9.2.1 Data Preprocessing *214*
9.3 The Bagging Voronoi Strategy *216*
9.4 Bagging Voronoi Clustering (BVClu) *218*
9.4.1 BVClu of the Telecom Data *221*
9.4.1.1 Setting the BVClu Parameters *221*
9.4.1.2 Results *223*
9.5 Bagging Voronoi Dimensional Reduction (BVDim) *223*
9.5.1 BVDim of the Telecom Data *225*
9.5.1.1 Setting the BVDim Parameters *225*
9.5.1.2 Results *227*
9.6 Bagging Voronoi Regression (BVReg) *231*
9.6.1 Covariate Information: The DUSAF Data *232*
9.6.2 BVReg of the Telecom Data *234*
9.6.2.1 Setting the BVReg Parameters *234*
9.6.2.2 Results *235*
9.7 Conclusions and Discussion *236*
References *239*

10 Nonparametric Inference for Spatiotemporal Data Based on Local Null Hypothesis Testing for Functional Data *242*
Alessia Pini and Simone Vantini
10.1 Introduction *242*
10.2 Methodology *244*
10.2.1 Comparing Means of Two Functional Populations *244*
10.2.2 Extensions *248*
10.2.2.1 Multiway FANOVA *249*
10.3 Data Analysis *250*
10.4 Conclusion and Future Works *256*
References *258*

11 Modeling Spatially Dependent Functional Data by Spatial Regression with Differential Regularization *260*
Mara S. Bernardi and Laura M. Sangalli
11.1 Introduction *260*
11.2 Spatial Regression with Differential Regularization for Geostatistical Functional Data *264*
11.2.1 A Separable Spatiotemporal Basis System *265*
11.2.2 Discretization of the Penalized Sum-of-Square Error Functional *268*
11.2.3 Properties of the Estimators *271*
11.2.4 Model Without Covariates *273*
11.2.5 An Alternative Formulation of the Model *274*
11.3 Simulation Studies *274*
11.4 An Illustrative Example: Study of the Waste Production in Venice Province *278*
11.4.1 The Venice Waste Dataset *278*
11.4.2 Analysis of Venice Waste Data by Spatial Regression with Differential Regularization *279*
11.5 Model Extensions *282*
References *283*

12 Quasi-maximum Likelihood Estimators for Functional Linear Spatial Autoregressive Models *286*
Mohamed-Salem Ahmed, Laurence Broze, Sophie Dabo-Niang, and Zied Gharbi
12.1 Introduction *286*
12.2 Model *288*
12.2.1 Truncated Conditional Likelihood Method *291*
12.3 Results and Assumptions *293*
12.4 Numerical Experiments *298*
12.4.1 Monte Carlo Simulations *298*
12.4.2 Real Data Application *305*
12.5 Conclusion *312*
12.A Appendix *313*
Proof of Proposition 12.A.1 *313*
Proof of Theorem 12.1 *314*
Proof of Theorem 12.2 *317*
Proof of Theorem 12.3 *319*
Proof of Lemma 12.A.2 *322*
Proof of Lemma 12.A.3 *322*
Proof of Lemma 12.A.5 *323*
References *325*

13 Spatial Prediction and Optimal Sampling for Multivariate Functional Random Fields *329*
Martha Bohorquez, Ramón Giraldo, and Jorge Mateu
13.1 Background *329*
13.1.1 Multivariate Spatial Functional Random Fields *329*
13.1.2 Functional Principal Components *330*
13.1.3 The Spatial Random Field of Scores *331*
13.2 Functional Kriging *332*
13.2.1 Ordinary Functional Kriging (OFK) *332*
13.2.2 Functional Kriging Using Scalar Simple Kriging of the Scores (FK_{SK}) *333*
13.2.3 Functional Kriging Using Scalar Simple Cokriging of the Scores (FK_{CK}) *333*
13.3 Functional Cokriging *336*
13.3.1 Cokriging with Two Functional Random Fields *336*
13.3.2 Cokriging with P Functional Random Fields *338*
13.4 Optimal Sampling Designs for Spatial Prediction of Functional Data *340*
13.4.1 Optimal Spatial Sampling for OFK *341*
13.4.2 Optimal Spatial Sampling for FK_{SK} *341*
13.4.3 Optimal Spatial Sampling for FK_{CK} *342*
13.4.4 Optimal Spatial Sampling for Functional Cokriging *343*
13.5 Real Data Analysis *344*
13.6 Discussion and Conclusions *348*
References *348*

Part III Spatio–Temporal Functional Data *351*

14 Spatio–temporal Functional Data Analysis *353*
Gregory Bopp, John Ensley, Piotr Kokoszka, and Matthew Reimherr
14.1 Introduction *353*
14.2 Randomness Test *355*
14.3 Change-Point Test *359*
14.4 Separability Tests *362*
14.5 Trend Tests *365*
14.6 Spatio–Temporal Extremes *369*
References *373*

15 A Comparison of Spatiotemporal and Functional Kriging Approaches *375*
Johan Strandberg, Sara Sjöstedt de Luna, and Jorge Mateu
15.1 Introduction *375*
15.2 Preliminaries *376*
15.3 Kriging *378*
15.3.1 Functional Kriging *378*
15.3.1.1 Ordinary Kriging for Functional Data *378*
15.3.1.2 Pointwise Functional Kriging *380*
15.3.1.3 Functional Kriging Total Model *381*
15.3.2 Spatiotemporal Kriging *382*
15.3.3 Evaluation of Kriging Methods *384*
15.4 A Simulation Study *385*
15.4.1 Separable *385*
15.4.2 Non-separable *390*
15.4.3 Nonstationary *391*
15.5 Application: Spatial Prediction of Temperature Curves in the Maritime Provinces of Canada *394*
15.6 Concluding Remarks *400*
References *400*

16 From Spatiotemporal Smoothing to Functional Spatial Regression: a Penalized Approach *403*
Maria Durban, Dae-Jin Lee, María del Carmen Aguilera Morillo, and Ana M. Aguilera
16.1 Introduction *403*
16.2 Smoothing Spatial Data via Penalized Regression *404*
16.3 Penalized Smooth Mixed Models *407*
16.4 P-spline Smooth ANOVA Models for Spatial and Spatiotemporal data *409*
16.4.1 Simulation Study *411*
16.5 P-spline Functional Spatial Regression *413*
16.6 Application to Air Pollution Data *415*
16.6.1 Spatiotemporal Smoothing *416*
16.6.2 Spatial Functional Regression *416*
Acknowledgments *421*
References *421*

Index *424*