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Mark Srednicki, *Physics Today*

Nobel Laureate Steven Weinberg combines exceptional physical insight with his gift for clear exposition, to provide a concise introduction to modern quantum mechanics, in this fully updated second edition of his successful textbook.

Now including six brand new sections covering key topics such as the rigid rotator and quantum key distribution, as well as major additions to existing topics throughout, this revised edition is ideally suited to a one-year graduate course or as a reference for researchers. Beginning with a review of the history of quantum mechanics and an account of classic solutions of the Schrödinger equation, before quantum mechanics is developed in a modern Hilbert space approach, Weinberg uses his remarkable expertise to elucidate topics such as Bloch waves and band structure, the Wigner-Eckart theorem, magic numbers, isospin symmetry, and general scattering theory. Problems are included at the ends of chapters, with solutions available for instructors at www.cambridge.org/9781107111660.

New for the second edition:

- The rigid rotator
- Van der Waals forces
- Rabi oscillators and Ramsey interferometers
- Open systems
- Time reversal of scattering processes
- Quantum key distribution

CAMBRIDGE
UNIVERSITY PRESS
www.cambridge.org

ISBN 978-1-107-11166-0



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