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Discrete data

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Often one is presented with numerical values of a function $f(x)$ at specified values of x . Experimental results are often presented in a table as a set of discrete data points. When data is presented in this way, the values of the function at points not given in the table must be found by some numerical technique.

Table 1.1 is a numerical representation of a function $f(x)$ at five different values of x .

x	$f(x)$
0.0	0.50
1.1	1.10
1.8	2.10
2.4	3.70
3.7	4.00

Table 1.1 Sample data table

Values of $f(x)$ at points x in the set $\{0, 1.1, 1.8, 2.4, 3.7\}$ are called *tabulated values*. For points $0.0 < x < 3.7$, values of $f(x)$ are called *interpolated values* of the data. Methods of finding interpolated values of $f(x)$ that we will develop in this chapter can also be used to predict extrapolated values of $f(x)$, but extrapolated values are often unreliable.

Graphical interpolation

A most straightforward approach to interpolation is to construct an approximate graph of the function and read the values of $f(x)$ from the graph.