Contents

1. Introduction 1

2. Quantisation of the Electromagnetic Field 7

2.1 Field Quantisation. 7
2.2 Fock or Number States. 10
2.3 Coherent States 12
2.4 Squeezed States 15
2.5 Two-Photon Coherent States 18
2.6 Variance in the Electric Field 20
2.7 Multimode Squeezed States. 22
2.8 Phase Properties of the Field. 23
Exercises 26

3. Coherence Properties of the Electromagnetic Field 29

3.1 Field-Correlation Functions 29
3.2 Properties of the Correlation Functions 31
3.3 Correlation Functions and Optical Coherence 32
3.4 First-Order Optical Coherence 34
3.5 Coherent Field 38
3.6 Photon Correlation Measurements. 39
3.7 Quantum Mechanical Fields. 41
3.7.1 Squeezed States 42
3.7.2 Squeezed Vacuum. 44
3.8 Phase-Dependent Correlation Functions 44
3.9 Photon Counting Measurements 46
3.9.1 Classical Theory 46
3.9.2 Constant Intensity 48
3.9.3 Fluctuating Intensity – Short-Time Limit 48
3.10 Quantum Mechanical Photon Count Distribution . . . 51
3.10.1 Coherent Light 52
3.10.2 Chaotic Light 52
3.10.3 Photo-Electron Current Fluctuations. 53
Exercises 55

4. Representations of the Electromagnetic Field 57

4.1 Expansion in Number States 57
4.2 Expansion in Coherent States 58
4.2.1 *P* Representation 58
a) Correlation Functions 60
b) Covariance Matrix 61
c) Characteristic Function 62
4.2.2 Wigner's Phase-Space Density 63
a) Coherent State 64
b) Squeezed State 64
c) Number State 64
4.2.3 *Q* Function 65
4.2.4 *R* Representation 66
4.2.5 Generalized *P* Representations 68
a) Number State 71
b) Squeezed State 71
4.2.6 Positive *P* Representation 71
Exercises 72

5. Quantum Phenomena in Simple Systems in Nonlinear Optics 73

5.1 Single-Mode Quantum Statistics 73
5.1.1 Degenerate Parametric Amplifier 73
5.1.2 Photon Statistics 75
5.1.3 Wigner Function 75
5.2 Two-Mode Quantum Correlations 77
5.2.1 Non-degenerate Parametric Amplifier 77
5.2.2 Squeezing 80
5.2.3 Quadrature Correlations and the Einstein– Podolsky–Rosen Paradox 81
5.2.4 Wigner Function 82
5.2.5 Reduced Density Operator 83
5.3 Quantum Limits to Amplification 85
5.4 Amplitude Squeezed State with Poisson Photon Number Statistics 86
Problems 89

6. Stochastic Methods 91

6.1 Master Equation 91
6.2 Equivalent c-Number Equations 98
6.2.1 Photon Number Representation 98
6.2.2 *P* Representation 99
6.2.3 Properties of Fokker–Planck Equations 101

6.2.4 Steady State Solutions – Potential Conditions . . 101
6.2.5 Time Dependent Solution 103
6.2.6 *Q* Representation 104
6.2.7 Wigner Function 106
6.2.8 Generalized *P* Represention 108
a) Complex *P* Representation 109
b) Positive *P* Representation 110
6.3 Stochastic Differential Equations 111
6.3.1 Use of the Positive *P* Representation 115
6.4 Linear Processes with Constant Diffusion 115
6.5 Two Time Correlation Functions in Quantum Markov Processes 117
6.5.1 Quantum Regression Theorem 118
6.6 Application to Systems with a *P* Representation 118
Exercises 119

7. Input–Output Formulation of Optical Cavities 121

7.1 Cavity Modes 121
7.2 Linear Systems 124
7.3 Two-Sided Cavity 126
7.4 Two Time Correlation Functions 127
7.5 Spectrum of Squeezing 129
7.6 Parametric Oscillator 129
7.7 Squeezing in the Total Field 132
7.8 Fokker–Planck Equation 132
Exercises 135

8. Generation and Applications of Squeezed Light 137

8.1 Parametric Oscillation and Second Harmonic Generation 137
8.1.1 Semi-classical Steady States and Stability Analysis 139
8.1.2 Parametric Oscillation 139
8.1.3 Second Harmonic Generation 140
8.1.4 Squeezing Spectrum 141
8.1.5 Parametric Oscillation 142
8.1.6 Experiments 143
8.2 Twin Beam Generation and Intensity Correlations . . . 146
8.2.1 Second Harmonic Generation 150
8.2.2 Experiments 152
8.2.3 Dispersive Optical Bistability 153
8.3 Applications of Squeezed Light 158
8.3.1 Interferometric Detection of Gravitational Radiation 158

8.3.2 Sub-Shot-Noise Phase Measurements 172
Exercises 174

9. Nonlinear Quantum Dissipative Systems 177

9.1 Optical Parametric Oscillator: Complex P Function . . 177
9.2 Optical Parametric Oscillator: Positive P Function . . 182
9.3 Quantum Tunnelling Time 186
9.4 Dispersive Optical Bistability 191
9.5 Comment on the Use of the Q and Wigner Representations 193
Exercises 193
9.A Appendix 194
9.A.1 Evaluation of Moments for the Complex P function for Parametric Oscillation (9.17) 194
9.A.2 Evaluation of the Moments for the Complex P Function for Optical Bistability (9.48) 195

10. Interaction of Radiation with Atoms 197

10.1 Quantization of the Electron Wave Field 197
10.2 Interaction Between the Radiation Field and the Electron Wave Field 199
10.3 Interaction of a Two-Level Atom with a Single Mode Field 204
10.4 Quantum Collapses and Revivals 205
10.5 Spontaneous Decay of a Two-Level Atom 206
10.6 Decay of a Two-Level Atom in a Squeezed Vacuum 208
10.7 Phase Decay in a Two-Level System 210
Exercises 211

11. Resonance Fluorescence 213

11.1 Master Equation 213
11.2 Spectrum of the Fluorescent Light 217
11.3 Photon Correlations 221
11.4 Squeezing Spectrum 225
Exercises 228

12. Quantum Theory of the Laser 229

12.1 Master Equation 229
12.2 Photon Statistics 232
12.2.1 Spectrum of Intensity Fluctuations 233
12.3 Laser Linewidth 235
12.4 Regularly Pumped Laser 236

12.A Appendix: Derivation of the Single-Atom Increment 240
Exercises 244

13. Intracavity Atomic Systems 245

13.1 Optical Bistability 245
13.2 Nondegenerate Four Wave Mixing 252
13.3 Experimental Results 258
Exercises 259

14. Bells Inequalities in Quantum Optics 261

14.1 The Einstein–Podolsky–Rosen (EPR) Argument . . . 261
14.2 Bell Inequalities and the Aspect Experiment 262
14.3 Violations of Bell's Inequalities Using a Parametric Amplifier Source 268
14.4 One-Photon Interference 273
Exercises 279

15. Quantum Nondemolition Measurements 281

15.1 Concept of a QND measurement 282
15.2 Back Action Evasion 284
15.3 Criteria for a QND Measurement 284
15.4 The Beam Splitter 287
15.5 Ideal Quadrature QND Measurements 290
15.6 Experimental Realisation 291
15.7 A Photon Number QND Scheme 294
Exercises 296

16. Quantum Coherence and Measurement Theory 297

16.1 Quantum Coherence 297
16.2 The Effect of Fluctuations 303
16.3 Quantum Measurement Theory 306
16.4 Examples of Pointer Observables 310
16.5 Model of a Measurement 311
Exercises 313

17. Atomic Optics 315

17.1 Young's Interference with Path Detectors 316
17.1.1 The Feynman Light Microscope 319
17.2 Atomic Diffraction by a Standing Light Wave 321
17.3 Optical Stern–Gerlach Effect 325
17.4 Quantum Non-Demolition Measurement of the Photon Number by Atomic Beam Deflection 330

17.5 Measurement of Atomic Position 334
17.5.1 Atomic Focussing and Contractive States. . . 337
Exercises 339
17.A Appendix. 339

References 341

Subject Index 347