

Contents

Prefaces

xi

I Linear Algebra Concepts and Matrix Decompositions

1

1 Vectors and Matrices in Data Mining and Pattern Recognition

3

1.1	Data Mining and Pattern Recognition	3
1.2	Vectors and Matrices	4
1.3	Purpose of the Book	9
1.4	Programming Environments	9
1.5	Floating Point Computations	9
1.6	Notation and Conventions	12

2 Vectors and Matrices

13

2.1	Matrix-Vector Multiplication	13
2.2	Matrix-Matrix Multiplication	14
2.3	Inner Product and Vector Norms	16
2.4	Matrix Norms	17
2.5	Linear Independence: Bases	19
2.6	The Rank of a Matrix	19

3 Linear Systems and Least Squares

21

3.1	LU Decomposition	21
3.2	Symmetric, Positive Definite Matrices	23
3.3	Perturbation Theory and Condition Number	24
3.4	Rounding Errors in Gaussian Elimination	24
3.5	Banded Matrices	26
3.6	The Least Squares Problem	28

4 Orthogonality

33

4.1	Orthogonal Vectors and Matrices	33
4.2	Elementary Orthogonal Matrices	36
4.3	Number of Floating Point Operations	41
4.4	Orthogonal Transformations in Floating Point Arithmetic	41

5 QR Decomposition

43

5.1	Orthogonal Transformation to Triangular Form	43
5.2	Solving the Least Squares Problem	46
5.3	Computing or Not Computing Q	47

5.4	Flop Count for QR Factorization	48
5.5	Error in the Solution of the Least Squares Problem	48
5.6	Updating the Solution of a Least Squares Problem	49
6	Singular Value Decomposition	53
6.1	The Decomposition	53
6.2	Fundamental Subspaces	57
6.3	Matrix Approximation	58
6.4	Principal Component Analysis	61
6.5	Solving Least Squares Problems	62
6.6	Condition Number and Perturbation Theory for the Least Squares Problem	64
6.7	Rank-Deficient and Underdetermined Systems	64
6.8	Computing the SVD	66
6.9	The Eigenvalue Decomposition of a Symmetric Matrix	66
6.10	Complete Orthogonal Decomposition	68
7	Reduced-Rank Least Squares Models	71
7.1	Truncated SVD: Principal Component Regression	73
7.2	A Krylov Subspace Method	75
8	Tensor Decomposition	85
8.1	Introduction	85
8.2	Basic Tensor Concepts	86
8.3	A Tensor SVD	88
8.4	Approximating a Tensor by HOSVD	90
9	Clustering and Nonnegative Matrix Factorization	95
9.1	The k -Means Algorithm	96
9.2	Nonnegative Matrix Factorization	99
10	Graphs and Matrices	105
10.1	Graphs and Adjacency Matrices	105
10.2	Connectedness and Reducibility	108
10.3	Graph Laplacians and Spectral Partitioning	109
10.4	Bipartite Graphs	115
II	Data Mining Applications	117
11	Classification of Handwritten Digits	119
11.1	Handwritten Digits and a Simple Algorithm	119
11.2	Classification Using SVD Bases	121
11.3	Tangent Distance	126
12	Text Mining	133
12.1	Preprocessing the Documents and Queries	134
12.2	The Vector Space Model	135
12.3	Latent Semantic Indexing	138
12.4	Clustering	142
12.5	Nonnegative Matrix Factorization	144

12.6	LGK Bidiagonalization	144
12.7	Average Performance	147
13	Page Ranking for a Web Search Engine	149
13.1	PageRank	150
13.2	Random Walk and Markov Chains	152
13.3	The Power Method for PageRank Computation	155
13.4	HITS	159
14	Automatic Key Word and Key Sentence Extraction	161
14.1	Saliency Score	161
14.2	Key Sentence Extraction from a Rank- k Approximation	164
15	Face Recognition Using Tensor SVD	169
15.1	Tensor Representation	169
15.2	Face Recognition	172
15.3	Face Recognition with HOSVD Compression	174
16	Spectral Graph Partitioning	177
16.1	Large and Sparse Laplacians	177
16.2	A Network of Political Blogs	178
16.3	Text Classification	181
16.4	Multiway Partitioning	186
III	Computing the Matrix Decompositions	187
17	Computing Eigenvalues and Singular Values	189
17.1	Perturbation Theory	190
17.2	The Power Method and Inverse Iteration	194
17.3	Similarity Reduction to Tridiagonal Form	196
17.4	The QR Algorithm for a Symmetric Tridiagonal Matrix	198
17.5	Computing the SVD	204
17.6	The Nonsymmetric Eigenvalue Problem	205
17.7	Sparse Matrices	207
17.8	The Arnoldi and Lanczos Methods	208
17.9	Software	214
	Bibliography	217
	Index	225