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It is hoped that whenever this software is available, the new chapter will encourage readers to investigate it in conjunction with it, and to learn more about mathematical functions and the operations that can be performed upon them. However, it was not intended to prepare some of the new material in this book. A reader should apply the software to some of the problems at the ends of chapters, though only to check the hand calculations that should first be carried out since they are essential when first learning about mathematics. Once the use of the software has been mastered, its power is such that it can then be applied to some realistic problems encountered in other courses which, if attempted by hand, could result in prohibitively long calculations.

As in previous editions, answers to odd numbered problems are given at the end of the book, some being given in more detail than others if a problem might cause difficulty.