

Contents

| | |
|---|-----------------|
| <i>Introduction</i> | <i>page</i> vii |
| 1 Preliminaries | 1 |
| 2 Symbolic Dynamics | 4 |
| 2.1 Topological pressure and variational principles | 5 |
| 2.2 Gibbs states, equilibrium states and potentials | 12 |
| 2.3 Perron–Frobenius operator | 26 |
| 2.4 Ionescu-Tulcea and Marinescu inequality | 31 |
| 2.5 Stochastic laws | 40 |
| 2.6 Analytic properties of pressure and the Perron–Frobenius operator | 43 |
| 2.7 The existence of eigenmeasures of the Conjugate Perron–Frobenius operator and of Gibbs states | 48 |
| 3 Hölder Families of Functions and F-Conformal Measures | 54 |
| 3.1 Summable Hölder families | 54 |
| 3.2 F -conformal measures | 57 |
| 4 Conformal Graph Directed Markov Systems | 62 |
| 4.1 Some properties of conformal maps in \mathbb{R}^d with $d \geq 2$ | 62 |
| 4.2 Conformal measures; Hausdorff and box dimensions | 71 |
| 4.3 Strongly regular, hereditarily regular and irregular systems | 87 |
| 4.4 Dimensions of measures | 90 |
| 4.5 Hausdorff, packing and Lebesgue measures | 94 |
| 4.6 Porosity of limit sets | 103 |
| 4.7 The associated iterated function system | 107 |
| 4.8 Refined geometry, F -conformal measures versus Hausdorff measures | 109 |
| 4.9 Multifractal analysis | 123 |

| | | |
|----------|---|-----|
| 5 | Examples of GDMSs | 136 |
| 5.1 | Examples of GDMSs in other fields of mathematics | 136 |
| 5.2 | Examples with special geometric features | 139 |
| 6 | Conformal Iterated Function Systems | 144 |
| 6.1 | The Radon-Nikodym derivative $\rho = \frac{d\mu}{dm}$ | 144 |
| 6.2 | Rate of approximation of the Hausdorff dimension by finite subsystems | 153 |
| 6.3 | Uniform perfectness | 156 |
| 6.4 | Geometric rigidity | 160 |
| 6.5 | Refined geometric rigidity | 165 |
| 7 | Dynamical Rigidity of CIFSs | 171 |
| 7.1 | General results | 171 |
| 7.2 | One-dimensional systems | 176 |
| 7.3 | Two-dimensional systems | 181 |
| 7.4 | Rigidity in dimension $d \geq 3$ | 195 |
| 8 | Parabolic Iterated Function Systems | 209 |
| 8.1 | Preliminaries | 209 |
| 8.2 | Topological pressure and associated parameters | 212 |
| 8.3 | Perron–Frobenius operator, semiconformal measures and Hausdorff dimension | 218 |
| 8.4 | The associated hyperbolic system. Conformal and invariant measures | 222 |
| 8.5 | Examples | 234 |
| 9 | Parabolic Systems: Hausdorff and Packing Measures | 238 |
| 9.1 | Preliminaries | 238 |
| 9.2 | The Case $d \geq 3$ | 240 |
| 9.3 | The plane case, $d = 2$ | 246 |
| 9.4 | Proofs of the main theorems | 255 |
| | <i>Appendix 1</i> Ergodic theory | 262 |
| | <i>Appendix 2</i> Geometric measure theory | 264 |
| | <i>Glossary of Notation</i> | 269 |
| | <i>Bibliography</i> | 272 |
| | <i>Index</i> | 280 |