

Contents

Preface	ix
Introduction	xi
Notes and references	xvi
Chapter 1 Mathematical background	1
1.1 Sets and functions	1
1.2 Some useful inequalities	3
1.3 Measures	6
1.4 Weak convergence of measures	13
1.5 Notes and references	16
Exercises	16
Chapter 2 Review of fractal geometry	19
2.1 Review of dimensions	19
2.2 Review of iterated function systems	29
2.3 Notes and references	39
Exercises	39
Chapter 3 Some techniques for studying dimension	41
3.1 Implicit methods	41
3.2 Box-counting dimensions of cut-out sets	51
3.3 Notes and references	56
Exercises	57
Chapter 4 Cookie-cutters and bounded distortion	59
4.1 Cookie-cutter sets	59
4.2 Bounded distortion for cookie-cutters	62
4.3 Notes and references	69
Exercises	69
Chapter 5 The thermodynamic formalism	71
5.1 Pressure and Gibbs measures	71
5.2 The dimension formula	75
5.3 Invariant measures and the transfer operator	79

5.4 Entropy and the variational principle	84
5.5 Further applications	88
5.6 Why 'thermodynamic' formalism?	92
5.7 Notes and references	94
Exercises	95
Chapter 6 The ergodic theorem and fractals	97
6.1 The ergodic theorem	97
6.2 Densities and average densities	102
6.3 Notes and references	111
Exercises	112
Chapter 7 The renewal theorem and fractals	113
7.1 The renewal theorem	113
7.2 Applications to fractals	123
7.3 Notes and references	128
Exercises	128
Chapter 8 Martingales and fractals	129
8.1 Martingales and the convergence theorem	129
8.2 A random cut-out set	136
8.3 Bi-Lipschitz equivalence of fractals	143
8.4 Notes and references	146
Exercises	146
Chapter 9 Tangent measures	149
9.1 Definitions and basic properties	149
9.2 Tangent measures and densities	155
9.3 Singular integrals	163
9.4 Notes and references	167
Exercises	167
Chapter 10 Dimensions of measures	169
10.1 Local dimensions and dimensions of measures	169
10.2 Dimension decomposition of measures	177
10.3 Notes and references	184
Exercises	184
Chapter 11 Some multifractal analysis	185
11.1 Fine and coarse multifractal theories	186
11.2 Multifractal analysis of self-similar measures	192

11.3 Multifractal analysis of Gibbs measures on cookie-cutter sets	201
11.4 Notes and references	204
Exercises	205
Chapter 12 Fractals and differential equations	207
12.1 The dimension of attractors	207
12.2 Eigenvalues of the Laplacian on regions with fractal boundary	223
12.3 The heat equation on regions with fractal boundary	230
12.4 Differential equations on fractal domains	236
12.5 Notes and references	244
Exercises	245
References	247
Index	253