

Contents

1	Steiner Triple Systems	1
1.1	The existence problem	1
1.2	$v \equiv 3 \pmod{6}$: The Bose Construction	4
1.3	$v \equiv 1 \pmod{6}$: The Skolem Construction	9
1.4	$v \equiv 5 \pmod{6}$: The $6n + 5$ Construction	14
1.5	Quasigroups with holes and Steiner triple systems	17
1.5.1	Constructing quasigroups with holes	17
1.5.2	Constructing Steiner triple systems using quasigroups with holes	22
1.6	The Wilson Construction	27
1.7	Cyclic Steiner triple systems	31
2	λ-Fold Triple Systems	37
2.1	Triple systems of index $\lambda > 1$	37
2.2	The existence of idempotent latin squares	39
2.3	2-Fold triple systems	42
2.3.1	Constructing 2-fold triple systems	42
2.4	$\lambda = 3$ and 6	47
2.5	λ -Fold triple systems in general.	50
3	Maximum Packings and Minimum Coverings	53
3.1	The general problem	53
3.2	Maximum packings.	58
3.3	Minimum coverings.	63
4	Kirkman Triple Systems	71
4.1	A recursive construction	71
4.2	Constructing pairwise balanced designs	79
5	Mutually Orthogonal Latin Squares	93
5.1	Introduction	93
5.2	The Euler and MacNeish Conjectures	97
5.3	Disproof of the MacNeish Conjecture	110
5.4	Disproof of the Euler Conjecture	113
5.5	Orthogonal latin squares of order $n \equiv 2 \pmod{4}$	116
5.6	Solution of the Euler Officer Problem.	93
5.7	MOLS(n) in standard form and complete sets.	96
5.8	The Euler Conjecture is a special case of the MacNeish Conjecture.	98
5.9	The direct product of MOLS: case 1.	107

6	Affine and Projective Planes	131
6.1	Affine planes.	131
6.2	Projective planes.	133
6.3	Connections between affine and projective planes.	135
6.4	Connection between affine planes and complete sets of MOLS(n).	137
6.5	Coordinatizing the affine plane.	140
7	Steiner Quadruple Systems	145
7.1	Introduction	145
7.2	Constructions of Steiner Quadruple Systems	153
7.3	The Stern and Lenz Lemma	158
7.4	The $(3v - 2u)$ -Construction	167
	Appendices	185
A	Cyclic Steiner Triple Systems	187
B	Answers to Selected Exercises	189