Contents

Preface to the Second Edition vii
Preface to the First Edition viii
Notation xv

CHAPTER 1
Fundamental Concepts of Group Theory 1
1.1. Binary Operations, Semigroups, and Groups 1
1.2. Examples of Groups 4
1.3. Subgroups and Cosets 8
1.4. Homomorphisms and Quotient Groups 17
1.5. Endomorphisms and Automorphisms 25
1.6. Permutation Groups and Group Actions 31

CHAPTER 2
Free Groups and Presentations 44
2.1. Free Groups 44
2.2. Presentations of Groups 50
2.3. Varieties of Groups 56

CHAPTER 3
Decompositions of a Group 63
3.1. Series and Composition Series 63
3.2. Some Simple Groups 71
3.3. Direct Decompositions 80

CHAPTER 4
Abelian Groups 93
4.1. Torsion Groups and Divisible Groups 93
4.2. Direct Sums of Cyclic and Quasicyclic Groups 98
4.3. Pure Subgroups and p-Groups 106
4.4. Torsion-Free Groups 114

CHAPTER 5
Soluble and Nilpotent Groups 121
5.1. Abelian and Central Series 121
5.2. Nilpotent Groups 129
5.3. Groups of Prime-Power Order 139
5.4. Soluble Groups 147

CHAPTER 6
Free Groups and Free Products 159
6.1. Further Properties of Free Groups 159
6.2. Free Products of Groups 167
6.3. Subgroups of Free Products 174
6.4. Generalized Free Products 184

CHAPTER 7
Finite Permutation Groups 192
7.1. Multiple Transitivity 192
7.2. Primitive Permutation Groups 197
7.3. Classification of Sharply k-Transitive Permutation Groups 203
7.4. The Mathieu Groups 208

CHAPTER 8
Representations of Groups 213
8.1. Representations and Modules 213
8.2. Structure of the Group Algebra 223
8.3. Characters 226
8.4. Tensor Products and Representations 235
8.5. Applications to Finite Groups 246

CHAPTER 9
Finite Soluble Groups 252
9.1. Hall π-Subgroups 252
9.2. Sylow Systems and System Normalizers 261
9.3. p-Soluble Groups 269
9.4. Supersoluble Groups 274
9.5. Formations 277

CHAPTER 10
The Transfer and Its Applications 285
10.1. The Transfer Homomorphism 285
10.2. Grün's Theorems 292
10.3. Frobenius's Criterion for p-Nilpotence 295
10.4. Thompson's Criterion for p-Nilpotence 298
10.5. Fixed-Point-Free Automorphisms 305

CHAPTER 11
The Theory of Group Extensions 310
11.1. Group Extensions and Covering Groups 310
11.2. Homology Groups and Cohomology Groups 326
11.3. The Gruenberg Resolution 333
11.4. Group-Theoretic Interpretations of the (Co)homology Groups 341

CHAPTER 12
Generalizations of Nilpotent and Soluble Groups 356
12.1. Locally Nilpotent Groups 356
12.2. Some Special Types of Locally Nilpotent Groups 363
12.3. Engel Elements and Engel Groups 369
12.4. Classes of Groups Defined by General Series 376
12.5. Locally Soluble Groups 381

CHAPTER 13
Subnormal Subgroups 385
13.1. Joins and Intersections of Subnormal Subgroups 385
13.2. Permutability and Subnormality 393
13.3. The Minimal Condition on Subnormal Subgroups 396
13.4. Groups in Which Normality Is a Transitive Relation 402
13.5. Automorphism Towers and Complete Groups 408

CHAPTER 14
Finiteness Properties 416
14.1. Finitely Generated Groups and Finitely Presented Groups 416
14.2. Torsion Groups and the Burnside Problems 422
14.3. Locally Finite Groups 429
14.4. 2-Groups with the Maximal or Minimal Condition 437
14.5. Finiteness Properties of Conjugates and Commutators 439

CHAPTER 15
Infinite Soluble Groups 450
15.1. Soluble Linear Groups 450
15.2. Soluble Groups with Finiteness Conditions on Abelian Subgroups 455

15.3. Finitely Generated Soluble Groups and the Maximal Condition on Normal Subgroups 461
15.4. Finitely Generated Soluble Groups and Residual Finiteness 470
15.5. Finitely Generated Soluble Groups and Their Frattini Subgroups 474

Bibliography 479

Index 491