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This preliminary chapter collects together material about functions of a complex variable. The chief purpose is to give a treatment of some special functions that are particularly needed in later chapters. For a comprehensive introduction to the theory of functions of a complex variable, see, for example, Ahlfors [3], Bak and Newman [7], Noguchi [44], or Palka [45].

1.1 Analytic Functions

Analytic functions are a particular subset of the class of all complex-valued functions of a complex variable. The real and imaginary parts of complex numbers are constantly required. We use the standard notation that if

$$z = x + iy, \quad i^2 = -1,$$

is a complex number, with x, y real numbers, then

$$x = \operatorname{Re}(z), \quad y = \operatorname{Im}(z).$$

We shall refer to the set of all complex numbers as the complex plane, and denote it by \mathbb{C} .

An alternative polar form is often used, using the modulus $|z|$ and argument $\theta = \arg(z)$:

$$|z| = \sqrt{x^2 + y^2}, \quad x = |z| \cos \theta, \quad y = |z| \sin \theta.$$