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* Supplementary dynamic geometry software activities using *Cabri Geometry II* and *Geometer's Sketchpad* are available at the website:
`http://www.stolaf.edu/ people/cederj/geotext/info.htm.`