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There are many excellent texts on elementary differential equations designed for the standard sophomore course. However, in spite of the fact that most courses are one semester in length, the texts have evolved into calculus-like presentations that include a large collection of methods and applications, packaged with student manuals, and Web-based notes, projects, and assignments. All of this comes in several hundred pages of text, with many figures. Most students do not have the time or desire to read voluminous texts and explore their supplements. The format of this differential equations book is different: it is a one-semester, brief treatment of the basic ideas, models, and solution methods. Its limited coverage places it somewhere between an outline and a detailed textbook. I have tried to write concisely, to the point, and in plain language. Many worked examples and exercises are included. A student who works through this primer will have the tools to go to the next level in applying differential equations to problems in engineering, science, and applied mathematics. It can give some instructors, who want more concise coverage, an alternative to existing texts.

The numerical solution of differential equations is a central activity in science and engineering, and it is absolutely necessary to teach students some aspects of scientific computation as early as possible. I tried to build in flexibility regarding a computer environment. The text allows students to use a calculator or a computer algebra system to solve some problems numerically and symbolically, and templates of MATLAB and Maple programs and commands are given in an appendix. The instructor can include as much of this, or as little of this, as he or she desires.

For many years I have taught this material to students who have had a standard three-semester calculus sequence. It was well received by those who