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Associated with each real valued function of several real variables is a collection of sets, called level sets, which are useful in studying qualitative properties of the function. Given a function $f: U \rightarrow \mathbb{R}$, where $U \subset \mathbb{R}^{n+1}$, its level sets are the sets $f^{-1}(c)$ defined, for each real number c , by

$$f^{-1}(c) = \{(x_1, \dots, x_{n+1}) \in U : f(x_1, \dots, x_{n+1}) = c\}$$

The number c is called the height of the level set, and $f^{-1}(c)$ is called the level set at height c . Since $f^{-1}(c)$ is the solution set of the equation $f(x_1, \dots, x_{n+1}) = c$, the level set $f^{-1}(c)$ is often described as "the set $\{(x_1, \dots, x_{n+1}) \in U : f(x_1, \dots, x_{n+1}) = c\}$ ".

The "level set" and "height" terminologies arise from the relation between the level sets of a function and its graph. The graph of a function $f: U \rightarrow \mathbb{R}$ is the subset of \mathbb{R}^{n+2} defined by

$$\text{graph}(f) = \{(x_1, \dots, x_{n+1}) \in \mathbb{R}^{n+2} : (x_1, \dots, x_{n+1}) \in U \\ \text{and } x_{n+2} = f(x_1, \dots, x_{n+1})\}.$$

For $c \geq 0$, the level set of f at height c is just the set of all points in the domain of f over which the graph is at distance c (see Figure 1.1). For $c < 0$, the level set of f at height c is just the set of all points in the domain of f under which the graph lies at distance $-c$.

For example, the level sets $f^{-1}(c)$ of the function $f(x_1, \dots, x_{n+1}) = x_1^2 + \dots + x_{n+1}^2$ are empty for $c < 0$, consist of a single point (the origin) if $c = 0$, and for $c > 0$ consist of two points if $n = 0$, circles centered at the origin with radius \sqrt{c} if $n = 1$, spheres centered at the origin with radius \sqrt{c} if $n = 2$, etc (see Figures 1.1 and 1.2).