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The reader should take notice of the following:

In Chapters I, II, III all terms denoting geometrical notions are to be understood in the Euclidean sense. In Chapter III those denoting notions of hyperbolic geometry are provided with the prefix *h*. In the following chapters terms denoting geometrical notions are to be understood in the sense of hyperbolic geometry. Those denoting Euclidean notions are provided with prefix *e*.

The values of square roots of positive numbers are always assumed to be positive.