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the book is all about.

Let π be a finite affine plane of order q^2 . An affine subplane of order q is said to be a 'Baer subplane'. Let D denote a set of $q + 1$ parallel classes such that for any two distinct affine points P and Q , such that the line ℓ_{PQ} joining them is an element of one of the parallel classes of D , there exists a Baer subplane π_{PQ} that contains P and Q such that the set of $q + 1$ parallel classes of π_{PQ} are those of D . In this situation, we call π a 'derivable' affine plane. Furthermore, D is called a 'derivation set'.

In the early 60's, T. G. Ostrom ([62], [61]) realized that when such a set D exists, a potentially new affine plane $\pi(D)$ may be constructed from π in the following manner:

The 'points' of $\pi(D)$ are the points of π and the 'lines' of $\pi(D)$ are the lines of π which are not in a class of D together with the Baer subplanes π_{PQ} .

A. A. Albert [1], showed that the derivation process applies to affine planes 'coordinatized' by finite fields K of order q^2 . Considering the affine plane as an analogue to the real affine plane, points are elements (x, y) of $K \times K$ and lines are given by equations $y = \alpha x + b$ and $x = c$ where juxtaposition denotes multiplication in K for all $m, b, c \in K$. The set D consists of the slopes (α) such that α is in the unique subfield of K of order q .