

CONTENTS

PREFACE	iii
1 A BRIEF OVERVIEW	1
2 PROJECTIVE GEOMETRIES	7
2.1 Projective and Affine Geometries.	9
2.2 Projective and Affine Planes.	14
3 BEGINNING DERIVATION	17
3.1 Affine Forms.	19
3.1.1 Affine Space as a Vector Space.	21
3.2 PG(3,q) and the Regulus.	21
3.2.1 The Standard Vector Space Form of a Regulus.	22
3.2.2 The Regulus Question.	23
3.3 Infinite Derivation.	23
4 SPREADS	27
4.1 Affine Planes with a Transitive Translation Group.	27
4.2 The Fundamental Theorem of Translation Planes.	31
4.3 The Basis Decomposition Theorem.	33
4.3.1 Some Examples.	37
4.3.2 Partial Spreads.	39
4.3.3 Derivable Partial Spread.	40

5	DERIVABLE NETS	41
5.1	Coordinatization.	42
5.1.1	Coordinates of a Net.	43
5.1.2	The Ternary Function.	43
5.1.3	Addition.	44
5.1.4	Multiplication.	44
5.1.5	Extension to an Affine Plane and Linearity.	45
5.1.6	The Question of Linearity.	45
5.2	Coordinates for Translation Planes and Their Duals.	47
5.2.1	Kernel Mappings and Kernel of the Quasifield.	50
5.2.2	Dual Affine Planes.	51
5.2.3	Dual Translation Planes and Derivation.	52
5.2.4	Chains and Semi-Translation Planes.	53
6	THE HUGHES PLANES	57
6.1	The Affine Hughes Planes.	60
7	DESARGUESIAN PLANES	65
7.1	Moufang Planes.	67
7.1.1	Coordinate Structures for Moufang Planes.	69
8	PAPPIAN PLANES	79
8.1	Reguli in Pappian Spreads.	80
9	CHARACTERIZATIONS OF GEOMETRIES	85
9.1	The Fundamental Theorem of Projective Geometry.	85
9.2	Projective and Affine Geometries.	88
10	DERIVABLE NETS AND GEOMETRIES	97
10.1	The Characterization Theorem.	102
10.1.1	Cofman's Theorem (for Derivable Nets).	104
10.1.2	A Quilt of Affine Geometries.	107
11	STRUCTURE THEORY FOR DERIVABLE NETS	113
11.1	The Collineation Group of a Derivable Net.	114
11.1.1	The Translation Group of a Derivable Net.	116
11.1.2	A Representation for the Translation Group.	118

11.1.3 The Group of Quasi-Skew and Skew Perspectives. . 122

11.2 The Structure Theorem for Derivable Nets. 126

11.3 The Answer to the Regulus Question. 127

11.4 The Solution to the Coordinates Problem. 127

12 DUAL SPREADS AND BAER SUBPLANES 131

12.1 Coordinate-Free Derivation and the Extension Question. . . . 137

12.1.1 Coordinate-Free Derivation. 141

13 DERIVATION AS A GEOMETRIC PROCESS 143

13.1 The Annihilator Mapping. 143

13.2 A ‘Geometric’ Interpretation. 145

14 EMBEDDING 149

14.1 Embedding Nets into Affine Space. 151

14.2 Embedding Nets into Projective Space. 154

14.3 Co-dimension-Two and Pseudo-Regulus Nets. 156

14.3.1 Embedded Nets Are Pseudo-Regulus Nets. 163

15 CLASSIFICATION OF SUBPLANE COVERED NETS 165

15.1 Sketch of the Embedding. 167

15.2 The Share Two Theorem. 168

15.3 The Derivable Net Substructures. 177

15.3.1 The Subnet Theorem. 183

15.4 The Parallel Classes Are Affine Spaces. 186

15.5 The Projective Space. 191

15.5.1 The Classification Theorem. 195

15.5.2 The Theorem on Pseudo-Regulus Nets. 198

16 SUBPLANE COVERED AFFINE PLANES 201

16.1 Regular Spreads. 201

16.2 The Affine Little Reidemeister Condition. 206

16.3 Embedded Nets. 210

16.4 Subplane Covered Translation Planes. 213

17 DIRECT PRODUCTS	215
17.1 The Reconstruction Theorem.	217
17.1.1 Desarguesian Products.	225
17.2 Left Spreads Produce Right Partial Spreads.	228
17.3 Left Spreads Produce Left Partial Spreads.	229
17.3.1 2-fold Products.	231
18 PARALLELISMS	233
18.1 Desarguesian Parallelisms.	233
18.2 When the Vector Space Is 4-dimensional.	235
18.3 Desarguesian Left Parallelisms Construct Right Spreads.	237
18.4 An Infinite Class of 'Derived' Parallelisms.	239
18.5 The Parallelisms in $PG(3, 2)$	243
19 PARTIAL PARALLELISMS WITH DEFICIENCY	245
19.1 Deficiency One.	247
19.2 Partial Dual Parallelisms.	249
20 BAER EXTENSIONS	251
20.1 Point-Baer Collineations.	251
20.2 Notes on the Non-Commutative Case.	256
20.2.1 Left and Right in Direct Product Nets.	259
20.2.2 When K Is K_0	261
20.3 Point-Baer Subplanes in Vector Space Nets.	262
20.3.1 Three Point-Baer Subplanes.	263
20.3.2 When There Are Two Point-Baer Subplanes.	268
20.3.3 The Space Is Decomposable by a Point-Baer Subplane	270
21 TRANSLATION PLANES ADMITTING BAER GROUPS	275
21.1 General Spreads with Baer Groups.	275
21.2 Translation Planes with Spreads in $PG(3, K)$	285
22 SPREADS COVERED BY PSEUDO-REGULI	293
22.1 Pseudo-Reguli.	293
22.2 Conical and Ruled Spreads over Skewfields.	300

23 CONICAL AND RULED PLANES OVER FIELDS	309
23.1 Conical Flocks.	309
23.2 Hyperbolic Flocks.	313
24 SPREADS WHICH ARE DUAL SPREADS	321
24.1 Chains and Semi-translation Planes Revisited.	326
25 PARTIAL FLOCKS OF DEFICIENCY ONE	329
25.1 Point-Baer Subplanes in Spreads of $PG(3, K)$	329
25.2 Deficiency-One Partial Conical Flocks.	331
25.3 Deficiency-One Partial Hyperbolic Flocks.	336
26 SKEW-HALL PLANES	341
26.1 Double-covers.	347
26.2 Postscript.	350
BIBLIOGRAPHY	353
INDEX	359