

# Contents

<i>Preface</i>	<i>page</i>	<i>xiii</i>
1 Basic Concepts in Probability and Statistics		1
1.1 Graphical Description of Data		2
1.2 Measures of Central Value: Mean, Median, and Mode		4
1.3 Measures of Variation: Percentile Ranges and Variance		6
1.4 Population versus a Sample		8
1.5 Elements of Probability Theory		8
1.6 Expectation		11
1.7 More Than One Random Variable		13
1.8 Independence		16
1.9 Estimating Population Quantities from Samples		18
1.10 Normal Distribution and Associated Theorems		20
1.11 Independence versus Zero Correlation		27
1.12 Further Topics		28
1.13 Conceptual Questions		29
2 Hypothesis Tests		30
2.1 The Problem		31
2.2 Introduction to Hypothesis Testing		33
2.3 Further Comments on the $t$ -test		40
2.4 Examples of Hypothesis Tests		43
2.5 Summary of Common Significance Tests		49
2.6 Further Topics		50
2.7 Conceptual Questions		51
3 Confidence Intervals		52
3.1 The Problem		53



3.2	Confidence Interval for a Difference in Means	53
3.3	Interpretation of the Confidence Interval	55
3.4	A Pitfall about Confidence Intervals	57
3.5	Common Procedures for Confidence Intervals	57
3.6	Bootstrap Confidence Intervals	64
3.7	Further Topics	67
3.8	Conceptual Questions	68
4	Statistical Tests Based on Ranks	69
4.1	The Problem	70
4.2	Exchangeability and Ranks	71
4.3	The Wilcoxon Rank-Sum Test	73
4.4	Stochastic Dominance	78
4.5	Comparison with the $t$ -test	79
4.6	Kruskal–Wallis Test	81
4.7	Test for Equality of Dispersions	83
4.8	Rank Correlation	85
4.9	Derivation of the Mean and Variance of the Rank Sum	88
4.10	Further Topics	92
4.11	Conceptual Questions	93
5	Introduction to Stochastic Processes	94
5.1	The Problem	95
5.2	Stochastic Processes	100
5.3	Why Should I Care if My Data Are Serially Correlated?	105
5.4	The First-Order Autoregressive Model	109
5.5	The AR(2) Model	117
5.6	Pitfalls in Interpreting ACFs	119
5.7	Solutions of the AR(2) Model	121
5.8	Further Topics	122
5.9	Conceptual Questions	124
6	The Power Spectrum	126
6.1	The Problem	127
6.2	The Discrete Fourier Transform	129
6.3	Parseval's Identity	133
6.4	The Periodogram	134
6.5	The Power Spectrum	135
6.6	Periodogram of Gaussian White Noise	138
6.7	Impact of a Deterministic Periodic Component	139



6.8	Estimation of the Power Spectrum	140
6.9	Presence of Trends and Jump Discontinuities	144
6.10	Linear Filters	146
6.11	Tying Up Loose Ends	150
6.12	Further Topics	152
6.13	Conceptual Questions	155
7	Introduction to Multivariate Methods	156
7.1	The Problem	157
7.2	Vectors	159
7.3	The Linear Transformation	160
7.4	Linear Independence	163
7.5	Matrix Operations	166
7.6	Invertible Transformations	168
7.7	Orthogonal Transformations	170
7.8	Random Vectors	172
7.9	Diagonalizing a Covariance Matrix	175
7.10	Multivariate Normal Distribution	178
7.11	Hotelling's T-squared Test	179
7.12	Multivariate Acceptance and Rejection Regions	181
7.13	Further Topics	182
7.14	Conceptual Questions	183
8	Linear Regression: Least Squares Estimation	185
8.1	The Problem	186
8.2	Method of Least Squares	188
8.3	Properties of the Least Squares Solution	192
8.4	Geometric Interpretation of Least Squares Solutions	196
8.5	Illustration Using Atmospheric CO <sub>2</sub> Concentration	199
8.6	The Line Fit	205
8.7	Always Include the Intercept Term	206
8.8	Further Topics	207
8.9	Conceptual Questions	209
9	Linear Regression: Inference	210
9.1	The Problem	211
9.2	The Model	212
9.3	Distribution of the Residuals	212
9.4	Distribution of the Least Squares Estimates	213
9.5	Inferences about Individual Regression Parameters	215



9.6	Controlling for the Influence of Other Variables	216
9.7	Equivalence to “Regressing Out” Predictors	218
9.8	Seasonality as a Confounding Variable	222
9.9	Equivalence between the Correlation Test and Slope Test	224
9.10	Generalized Least Squares	225
9.11	Detection and Attribution of Climate Change	226
9.12	The General Linear Hypothesis	233
9.13	Tying Up Loose Ends	234
9.14	Conceptual Questions	236
10	Model Selection	237
10.1	The Problem	238
10.2	Bias–Variance Trade off	240
10.3	Out-of-Sample Errors	243
10.4	Model Selection Criteria	245
10.5	Pitfalls	249
10.6	Further Topics	253
10.7	Conceptual Questions	254
11	Screening: A Pitfall in Statistics	255
11.1	The Problem	256
11.2	Screening <i>iid</i> Test Statistics	259
11.3	The Bonferroni Procedure	262
11.4	Screening Based on Correlation Maps	262
11.5	Can You Trust Relations Inferred from Correlation Maps?	265
11.6	Screening Based on Change Points	265
11.7	Screening with a Validation Sample	268
11.8	The Screening Game: Can You Find the Statistical Flaw?	268
11.9	Screening Always Exists in Some Form	271
11.10	Conceptual Questions	272
12	Principal Component Analysis	273
12.1	The Problem	274
12.2	Examples	276
12.3	Solution by Singular Value Decomposition	283
12.4	Relation between PCA and the Population	285
12.5	Special Considerations for Climate Data	289
12.6	Further Topics	295
12.7	Conceptual Questions	297



13	Field Significance	298
13.1	The Problem	299
13.2	The Livezey–Chen Field Significance Test	303
13.3	Field Significance Test Based on Linear Regression	305
13.4	False Discovery Rate	310
13.5	Why Different Tests for Field Significance?	311
13.6	Further Topics	312
13.7	Conceptual Questions	312
14	Multivariate Linear Regression	314
14.1	The Problem	315
14.2	Review of Univariate Regression	317
14.3	Estimating Multivariate Regression Models	320
14.4	Hypothesis Testing in Multivariate Regression	323
14.5	Selecting X	324
14.6	Selecting Both X and Y	328
14.7	Some Details about Regression with Principal Components	331
14.8	Regression Maps and Projecting Data	332
14.9	Conceptual Questions	333
15	Canonical Correlation Analysis	335
15.1	The Problem	336
15.2	Summary and Illustration of Canonical Correlation Analysis	337
15.3	Population Canonical Correlation Analysis	343
15.4	Relation between CCA and Linear Regression	347
15.5	Invariance to Affine Transformation	349
15.6	Solving CCA Using the Singular Value Decomposition	350
15.7	Model Selection	357
15.8	Hypothesis Testing	359
15.9	Proof of the Maximization Properties	362
15.10	Further Topics	364
15.11	Conceptual Questions	364
16	Covariance Discriminant Analysis	366
16.1	The Problem	367
16.2	Illustration: Most Detectable Climate Change Signals	370
16.3	Hypothesis Testing	378
16.4	The Solution	382
16.5	Solution in a Reduced-Dimensional Subspace	388
16.6	Variable Selection	392



16.7	Further Topics	395
16.8	Conceptual Questions	398
17	Analysis of Variance and Predictability	399
17.1	The Problem	400
17.2	Framing the Problem	401
17.3	Test Equality of Variance	403
17.4	Test Equality of Means: ANOVA	404
17.5	Comments about ANOVA	406
17.6	Weather Predictability	407
17.7	Measures of Predictability	411
17.8	What Is the Difference between Predictability and Skill?	414
17.9	Chaos and Predictability	416
17.10	Conceptual Questions	417
18	Predictable Component Analysis	418
18.1	The Problem	419
18.2	Illustration of Predictable Component Analysis	422
18.3	Multivariate Analysis of Variance	424
18.4	Predictable Component Analysis	427
18.5	Variable Selection in PrCA	430
18.6	PrCA Based on Other Measures of Predictability	432
18.7	Skill Component Analysis	435
18.8	Connection to Multivariate Linear Regression and CCA	437
18.9	Further Properties of PrCA	439
18.10	Conceptual Questions	445
19	Extreme Value Theory	446
19.1	The Problem and a Summary of the Solution	447
19.2	Distribution of the Maximal Value	453
19.3	Maximum Likelihood Estimation	459
19.4	Nonstationarity: Changing Characteristics of Extremes	463
19.5	Further Topics	466
19.6	Conceptual Questions	467
20	Data Assimilation	468
20.1	The Problem	469
20.2	A Univariate Example	469
20.3	Some Important Properties and Interpretations	473
20.4	Multivariate Gaussian Data Assimilation	475



20.5	Sequential Processing of Observations	477
20.6	Multivariate Example	478
20.7	Further Topics	481
20.8	Conceptual Questions	487
21	Ensemble Square Root Filters	489
21.1	The Problem	490
21.2	Filter Divergence	497
21.3	Monitoring the Innovations	499
21.4	Multiplicative Inflation	500
21.5	Covariance Localization	503
21.6	Further Topics	507
21.7	Conceptual Questions	509
<i>Appendix</i>		510
A.1	Useful Mathematical Relations	510
A.2	Generalized Eigenvalue Problems	511
A.3	Derivatives of Quadratic Forms and Traces	512
<i>References</i>		514
<i>Index</i>		523