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The goal of this textbook is to introduce the mathematical theory of incompressible fluid mechanics and its applications. It grew out of lectures for a new course on the subject given by SIAM to final-year undergraduate as well as graduate students in autumn 2010. The basic material for the lectures was drawn from lecture notes on 'Ideal Fluid Mechanics' devised by Frank and lecture notes on 'Viscous Fluid Mechanics' devised by Drew. In October 2013, David Traub from Cambridge University Press suggested these lecture notes might be the seed for a textbook. After some delay, we all met in autumn 2015, and the plans and structure for the book were laid. Once writing was in full swing, the book grew and developed substantially, and slightly transformed its character. However, we hope the character of the book retains/includes the following essential themes. It is accessible, comprehensive, mathematical, practical, engaging and useful. Let us briefly expand on these. We attempt to take interested students on a full journey. That journey starts with the very basic notion of how to represent the flow, carries the reader through analytical solutions and/or pragmatic approximations to practical flow scenarios, problems and applications, and ends at/on the issue of the global existence of solutions to the fluid equations themselves. The practical and mathematical go hand in hand, and a central theme of the book is the rigorous pursuit of the underlying mathematics and mathematical questions to obtain analytical solutions for the applied flow scenarios concerned. Another complementary and distinguishing theme is the plethora of extensive exercises we have included. These are styled progressively and aimed to complement the main theory and examples in the respective chapters.

We believe that Part I of the book (Inviscid Flow) is suitable for penultimate-year or final-year undergraduate students, while Part II (Viscous Flow) is suitable for final-year undergraduate and graduate students. This of course depends on mathematical background and knowledge. We include here mathematics, physics, science and engineering students, who are our target audience. We hope such students use the book as an introduction, springboard and long-term reference for their fluid mechanics knowledge and experience.

Many classical and contemporary classical textbooks have been invaluable in our preparation and exposition. Their influence is discernable throughout parts of the book. In particular, in general throughout, we relied on the classical textbooks by Batchelor, Currie, Kundu and Cohen, Lamb, Landau and Lifshitz, Lighthill, Paoletti, Tritton and Van Dyke, as well as some classical, more contemporary ones (e.g. Childress, Chorin and Marsden, Majda and Bertozzi) and Ockendon and Ockendon.