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B Bibliography	Number Domains $\mathbb{Z}, \mathbb{Q}, \mathbb{R}$, <i>pi</i> , <i>euler</i> , <i>goldenRatio</i> , <i>goldenAngle</i> , <i>classicPi</i> , <i>firstOrderTaylor</i> , <i>lastOrderTaylor</i>	333
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III Topology and Calculus

Limits and Topology, Differentiability, Inverse and Implicit Functions, Integration, Fubini and Changing Variables, Vector Fields, Fixpoints, Main Theorem of ODEs

IV Selected Higher Subjects

Numerics, Probability and Statistics, Splines, Fourier, Wavelets, Fractals, Neural Nets, Global Coordinates and Manifolds, Categories, Lambda Calculus