Contents

Preface — VII

1 The classical harmonic oscillator — 1

2 Lagrangian and Hamiltonian mechanics — 9
2.1 Introduction — **9**
2.2 State space and Lagrangians — **9**
2.3 Phase space and Hamiltonians — **46**
2.4 Segue — **64**

3 The formalism of quantum mechanics: motivation — 67
3.1 Introduction — **67**
3.2 States, observables and dynamics — **68**
3.3 Mechanics and Hilbert spaces — **74**

4 Physical background — 79
4.1 Introduction — **79**
4.2 Electromagnetic radiation — **80**
4.3 Blackbody radiation and the photoelectric effect — **96**
4.4 Two-slit experiments — **121**

5 Synopsis of self-adjoint operators, spectral theory and Stone's theorem — 129
5.1 Introduction — **129**
5.2 Unbounded self-adjoint operators — **131**
5.3 $H_B = \frac{1}{2m}P^2 + \frac{m\omega^2}{2}Q^2$ — **142**
5.4 The spectrum — **155**
5.5 The spectral theorem — **165**
5.6 Stone's theorem — **180**

6 The postulates of quantum mechanics — 189
6.1 Introduction — **189**
6.2 The postulates — **189**
6.3 Uncertainty relations — **218**
6.4 Schrödinger and Heisenberg pictures — **225**

7 Canonical quantization — 231
7.1 $PQ - QP = \frac{h}{2\pi i}$ — **231**
7.2 Heisenberg algebras and Heisenberg groups — **242**
7.3 The free quantum particle — **272**

7.4 The quantum harmonic oscillator — 286
7.4.1 Introduction — 286
7.4.2 The classical limit problem — 290
7.4.3 The propagator — 303

8 Path integral quantization — 311
8.1 Motivation — 311
8.2 Path integral for the free quantum particle — 321
8.3 Path integral for the harmonic oscillator — 325

9 Sketches of some rigorous results — 339
9.1 Introduction — 339
9.2 Self-adjointness of $-\frac{\hbar^2}{2m}\Delta + V$ — 343
9.3 Brownian motion and Wiener measure — 358
9.4 Analytic continuation — 383

10 Fermionic and supersymmetric harmonic oscillators — 391
10.1 The Stern–Gerlach experiment and spin one-half — 391
10.2 Anticommutation relations and the fermionic harmonic oscillator — 407
10.3 $N = 2$ supersymmetry and the harmonic oscillator — 425
10.4 $N = 2$ supersymmetry and Hodge theory — 440

A Gaussian integrals — 455

B Morse lemma — 465

C Stationary phase approximation — 473

D Tangent and cotangent bundles — 481

E Poisson and wave equations — 483

F Carathéodory procedure — 485

G Schwartz space, Fourier transform, distributions and Sobolev spaces — 487
G.1 Introduction — 487
G.2 Dimension 1 — 487
G.3 Heat kernel — 497
G.4 Dimension n — 502

H Stieltjes integrals — 511
H.1 Introduction — **511**
H.2 Riemann–Stieltjes integrals — **511**
H.3 Lebesgue–Stieltjes integrals — **512**

I Unitary representations and Schur's lemma — 517

J Semigroups of operators — 521

K Hilbert space tensor products — 529

Bibliography — 531

Index — 543