STIME STORES

CONTENTS

3

3

5

5

6

8

8

9

10

11

12

12

13

15

16

17

19

Fundamental concepts and equations

- 1.1 Some mathematical concepts and notation
 - 1.1.1 Basic notation
 - 1.1.2 Some useful inequalities in $I\!\!R^N$
 - 1.1.3 Differential operators
 - 1.1.4 Gronwall's lemma
 - 1.1.5 Implicit functions

Transformations of Cartesian coordinates 1.1.6Hölder-continuous and Lipschitz functions 1.1.7 The symbols "o" and "O" 1.1.8 Partitions of unity 1.1.9 1.1.10 Measure 1.1.11 Description of the boundary 1.1.12 Measure on the boundary of a domain 1.1.13 Classical Green's theorem 1.1.14 Lebesgue spaces 1.1.15 Lebesgue's points 1.1.16 Absolutely continuous functions 1.1.17 Absolute continuity of integrals with respect to measurable subsets 1.1.18 Some theorems from integration theory Governing equations and relations of gas dynamics 1.2 Description of the flow 1.2.1The transport theorem 1.2.2The continuity equation 1.2.3

1.2.4	The equations of motion	19
1.2.5	The law of conservation of the moment of	
	momentum. Symmetry of the stress tensor	21
1.2.6	Inviscid and viscous fluids	21
1.2.7	The energy equation	22
1.2.8	The second law of thermodynamics and the	
	entropy	22
1.2.9	Principle of material frame indifference	23
1.2.10	Newtonian fluids	24
1.2.11	Conservative and dissipation form of the energy	
	equation for Newtonian fluids	24
1.2.12	Entropy form of the energy equation for	
	Newtonian fluids	25

2

		1.2.13	Some consequences of the Clausius–Duhem	
			inequality	25
		1.2.14	Equations of state	26
		1.2.15	Adiabatic flow of a perfect inviscid gas	27
		1.2.16	Compressible Euler equations	28
		1.2.17	Compressible Navier–Stokes equations for a	
			perfect viscous gas	28
		1.2.18	Barotropic flow of a viscous gas	29
		1.2.19	Speed of sound	30
		1.2.20	Simplified models	30
		1.2.21	Initial and boundary conditions	31
	1.3	Some	advanced mathematical concepts and results	32
		1.3.1	Spaces of Hölder-continuous and continuously	
			differentiable functions	33
		1.3.2	Young's functions, Jensen's inequality	33
		1.3.3	Orlicz spaces	34
		1.3.4	Distributions	35
		1.3.5	Sobolev spaces	40
		1.3.6	Homogeneous Sobolev spaces	47
		1.3.7	Tempered distributions	50
		1.3.8	Radon measure and representation of $C_B(\Omega)^*$	52
		1.3.9	Functions of bounded variation	52
		1.3.10	Functions with values in Banach spaces	53
		1.3.11	Sobolev imbeddings of abstract spaces	57
		1.3.12	Some compactness results	58
	1.4	Survey	y of concepts and results from functional analysis	60
		1.4.1	Linear vector spaces	60
		1.4.2	Topological linear spaces	60
		1.4.3	Metric linear space	62
		1.4.4	Normed linear space	62
		1.4.5	Duals to Banach spaces and weak(-*) topologies	64
		1.4.6	Riesz representation theorem	68
		1.4.7	Operators	68
		1.4.8	Elements of spectral theory	70
		1.4.9	Lax–Milgram lemma	70
		1.4.10	Imbeddings	71
		1.4.11	Solution of nonlinear operator equations	71
,	The	orotics	al regults for the Fuler equations	74
	21	Hyper	bolic systems and the Euler equations	74
	2.1	211	Zero-viscosity Burgers equation	75
		21.2	One-dimensional Euler equations	76
		2.1.2	Lagrangian mass coordinates	76
		2.1.0	Symmetrizable systems	77
		2.1.4	Symmetrizable Systems	11

	915	Matrix form of the newstorn		77
	2.1.0	The Fuler equations of an inviscid ras		78
22	Evicto	ance of smooth solutions		70
2.2	2 9 1	Hyperbolic systems and characteristics		70
	2.2.1	Cauchy problem for system of conservation laws		80
	2.2.2	Linear scalar equation		81
	2.2.0	Solution of a linear system		82
	2.2.4	Nonlinear scalar equation		82
	2.2.0	Diston problem		81
	2.2.0	Complementary Diemann inverients		81
	2.2.1	Solution of the nisten problem		85
	2.2.0	Couchy problem for a symmetric hyperbolic		00
	2.2.9	Cauchy problem for a symmetric hyperbolic		80
	0 0 10	Amminations		00
	2.2.10	Enistence of enpressions		90
	2.2.11	Existence of approximations		90
	2.2.12	Energy estimate		91
	2.2.13	Convergence of approximations to a generalized		00
	0014	Solution		92
	2.2.14	Regularity of the generalized solution		92
	2.2.15	Quasilinear system		94
	2.2.16	Local existence for a quasilinear system		95
	2.2.17	Second grade approximations		95
	2.2.18	Higher order energy estimates		95
	2.2.19	Convergence of approximations		97
	2.2.20	Uniqueness		98
	2.2.21	Local existence for equations of an isentropic		~~~
		ideal gas	7	99
	2.2.22	Existence of global smooth solutions for		
		nonlinear hyperbolic systems		100
	2.2.23	2×2 system of conservation laws, Riemann		275.5
		invariants		100
	2.2.24	Plane wave solutions		103
	2.2.25	Plane waves for the Euler system in $2D$		104
2.3	Weak	solutions		106
	2.3.1	Blow up of classical solutions		107
	2.3.2	Generalized formulation for systems of		
		conservation laws		108
	2.3.3	Piecewise smooth solutions		108
	2.3.4	Entropy condition		110
	2.3.5	Physical entropy		112
	2.3.6	General parabolic approximation and the		
		entropy condition		113
	2.3.7	Entropy for a general scalar conservation law		115

xiii

		2.3.8	Entropy for a 2×2 system of conservation laws	
			in 1D	117
		2.3.9	Entropy function for a p -system	118
		2.3.10	Riemann problem	118
		2.3.11	Riemann problem for 2×2 is entropic gas	
			dynamics equations	120
		2.3.12	Existence and uniqueness of admissible weak	28
			solution for a scalar conservation law	125
		2.3.13	Plane waves admitting discontinuities	125
		2.3.14	Existence of solutions to the 2×2 Euler system	
			for an isentropic gas	125
		2.3.15	Lax–Friedrichs difference approximations	128
		2.3.16	Existence of approximations	129
		2.3.17	Invariant regions for Riemann invariants	129
		2.3.18	Compactness argument	130
		2.3.19	Characterization of the weak limit by Young	
			measure	132
		2.3.20	Div-curl lemma and Tartar's commutation	
			relation	134
		2.3.21	Existence of weak entropy-entropy flux pairs	135
		2.3.22	Localization of supp ν	138
		2.3.23	Approximative limit is an admissible solution	144
		2.3.24	Global existence for general systems in one	1.
			dimension	145
	2.4	Final	comments	146
		2.4.1	Local existence results	146
		2.4.2	Global smooth solutions	147
		2.4.3	Blow up and the lifespan of smooth solution	148
		2.4.4	Global weak solutions for multidimensional	
			Euler equations	150
		2.4.5	Riemann problem	151
		2.4.6	Euler equations with source terms	152
		2.4.7	Comments on the 2×2 Euler system for an	
			isentropic fluid	152
		2.4.8	Euler equations for a nonisentropic fluid	154
3	Som	e mat	hematical tools for compressible flows	155
- 2	3.1	Renor	malized solutions of the steady continuity	
		equati	on	155
		3.1.1	Friedrichs' lemma about commutators	155
		3.1.2	Continuity equation and its prolongation	158
		3.1.3	Renormalized solutions of the continuity	
			equation	159
	3.2	Vector	fields with summable divergence	163

	3.3	The e	equation div $v = f$	165
		3.3.1	Bounded domains	166
		3.3.2	Exterior domains	176
		3.3.3	Domains with noncompact boundaries	178
	3.4	Some	results for monotone and convex operators	183
		3.4.1	Some results from convex analysis	183
		3.4.2	Some results from monotone operators	186
L	Wea	ak solu	utions for steady Navier-Stokes equations	
	of co	ompre	essible barotropic flow	189
	4.1	Form	ulation of problems in bounded and exterior	
		doma	ins and main results	189
		4.1.1	Definition of weak solutions	190
		4.1.2	Existence of weak solutions	192
		4.1.3	Exterior domains	193
	4.2	Heuri	stic approach	194
		4.2.1	Estimates due to the energy inequality and	
			improved estimates of density	194
		4.2.2	Limit passage	195
		4.2.3	Effective viscous flux	196
		4.2.4	Strong convergence of density – Lions' approach	197
		4.2.5	Strong convergence of density – Feireisl's	
			approach	198
		4.2.6	Remarks to approximations	199
	4.3	Appro	oximations in bounded domains	200
		4.3.1	First level approximation – artificial pressure	200
		4.3.2	Second level approximation – relaxation in the	
			continuity equation	202
		4.3.3	Third level approximation – relaxed continuity	
			equation with dissipation	203
	4.4	Effect	ive viscous flux	204
		4.4.1	Riesz operators	205
		4.4.2	Div–curl lemma	206
		4.4.3	Commutator lemma	207
		4.4.4	Effective viscous flux	208
	4.5	Neum	ann problem for the Laplacian	211
		4.5.1	Existence, uniqueness and regularity	211
		4.5.2	Eigenvalue problem	212
	4.6	Relax	ed continuity equation with dissipation	212
		4.6.1	Statement of the problem and results	212
		4.6.2	Estimates for the Leray–Schauder fixed points	213
		4.6.3	Homotopy of compact transformations	215
		4.6.4	Nonnegativity of the density	216
	4.7	The L	lamé system	216

xv

	4.7.1 Existence, uniqueness and regularity			217	
	4.7.2 Eigenvalue problem			217	-
4.8	Complete system with dissipation in the relaxed				
	continuity equation and with artificial pressure			218	
	4.8.1 Existence of solutions			218	
	4.8.2 Estimates independent of dissipation			222	
4.9	Complete system with relaxed continuity equation and				
	with artificial pressure			223	
	4.9.1 Vanishing dissipation limit			224	
	4.9.2 Effective viscous flux			225	
	4.9.3 Renormalized continuity equation with powers			226	
	4.9.4 Strong convergence of the density			230	
	4.9.5 Equation of momentum, energy inequality and				
	estimates independent of the relaxation				
	parameter			231	
4.10	Complete system with artificial pressure			231	
	4.10.1 Vanishing relaxation limit			232	
	4.10.2 Effective viscous flux			233	
	4.10.3 Renormalized continuity equation with powers			234	
	4.10.4 Strong convergence of the density			235	
	4.10.5 Momentum equation			236	
	4.10.6 Energy inequality and estimates independent of				
	artificial pressure		1	236	
4.11	Complete system of a viscous barotropic gas		1	239	
	4.11.1 Vanishing artificial pressure limit			239	
	4.11.2 Effective viscous flux			241	
	4.11.3 Boundedness of oscillations of density sequence			241	
	4.11.4 Renormalized continuity equation			243	
	4.11.5 Strong convergence of the density	0.000		244	
4.12	Approximations in an exterior domain			245	
	4.12.1 Relaxation on invading domains			245	
4.13	Complete system with relaxed continuity equation on				
	an exterior domain			247	
	4.13.1 Some equivalence inequalities			247	
	4.13.2 Bounds due to the energy inequality			247	
	4.13.3 Estimates independent of invading domains and				
	relaxation			248	
4.14	Existence of weak solutions in exterior domains			254	
	4.14.1 Vanishing relaxation limit			254	
	4.14.2 Effective viscous flux and renormalized				
	continuity equation			255	
4.15	Existence of weak solutions in bounded and in exterior				
	Lipschitz domains			259	

xvii

4.16	Existence of weak solutions in domains with		
	noncompact boundaries		261
	4.16.1 Formulation of the problem, fluxes		262
	4.16.2 Main results		264
	4.16.3 Domains with conical or superconical exits		265
	4.16.4 Domains with cylindrical or subconical exits		268
4.17	Further results, comments and bibliographic remarks		268
	4.17.1 Weak compactness		268
	4.17.2 Bounded domains		269
	4.17.3 Exterior domains	100 1 m	274
	4.17.4 Domains with noncompact boundaries		275
	4.17.5 Flow of mixtures		278

5 Strong solutions for steady Navier-Stokes equations

,	Dure	ing soi	futions for steady mavier Stokes equations			
	of c	ompre	essible barotropic flow and small data			279
	5.1	Notat	ion and main results	2.3,1		279
		5.1.1	Formulation of the problem			279
		5.1.2	Existence theorem in a bounded domain			280
		5.1.3	Functional spaces for exterior domains			280
		5.1.4	Existence theorems in exterior domains			281
	5.2	Heuris	stic approach			282
		5.2.1	Perturbations and linearization of the problem			282
		5.2.2	Helmholtz decomposition and effective viscous	1		
			flux		1	283
		5.2.3	Existence theorem for the linearized system		14	285
	5.3	Auxili	iary linear problems			285
		5.3.1	Neumann problem for the Laplacian			286
		5.3.2	Helmholtz decomposition			286
		5.3.3	Dirichlet problem for the Laplacian			287
		5.3.4	Stokes and Oseen problems			287
		5.3.5	Steady transport equation			289
	5.4	The li	nearized system			290
	5.5	The fu	ully nonlinear system			292
		5.5.1	The case of zero velocity at infinity			292
		5.5.2	The case of nonzero velocity at infinity			295
	5.6	Biblio	graphic remarks			296
		5.6.1	Bounded domains			296
		5.6.2	Exterior domains			297
;	Som	ne mat	hematical tools for nonsteady equations		-	300
	6.1	Some	auxiliary results from functional analysis			300
		6.1.1	Continuous functions with values in L^q_{weak}			300
		6.1.2	The time and space mollifiers			303
		6.1.3	Local weak compactness in unbounded domains			304
	6.2	Renor	malized solutions of the continuity equation			304

xviii

	6.2.1	Friedrichs' lemma about commutators	304
	6.2.2	Continuity equation and its prolongation	306
	6.2.3	Renormalized continuity equation	307
	6.2.4	Strong continuity of the density	310
Wea	ak solu	itions for nonsteady Navier–Stokes	
equ	ations	of compressible barotropic flow	312
7.1	Form	ulation of problems and main results	312
	7.1.1	Definition of weak solutions	313
	7.1.2	Existence in bounded domains	318
	7.1.3	Existence in exterior domains	320
7.2	Linea	r momentum and total energy	321
	7.2.1	Linear momentum	321
	7.2.2	Total energy	322
7.3	Heuri	stic approach	324
	7.3.1	Compactness of weak solutions	324
	7.3.2	Estimates due to the energy inequality	325
	7.3.3	Improved estimate of the density	325
	7.3.4	Limit passage	326
	7.3.5	Effective viscous flux	326
	7.3.6	Strong convergence of density – Lions' approach	327
	7.3.7	Strong convergence of density – Feireisl's	200
	790	approach	328
7 1	1.3.8	Remarks on approximations	329
1.4	Appro	Einst level annuing tions	330
	7.4.1	First level approximations – artificial pressure	330
	1.4.2	equation with dissipation	222
	743	Third level approximation - Galerkin method	335
75	Effect	ive viscous flux	338
7.6	Conti	nuity equation with dissipation	343
	7.6.1	Regularity for the parabolic Neumann problem	343
	7.6.2	Continuity equation with dissipation	345
	7.6.3	Construction of a solution – Galerkin method	346
	7.6.4	Regularity of solutions	348
	7.6.5	Boundedness from below and from above	348
	7.6.6	L^2 -estimates	349
	7.6.7	L^2 -estimate of differences	350
	7.6.8	A renormalized inequality with dissipation	351
7.7	Galer	kin approximation of the system with dissipation	
	in the	continuity equation and with artificial pressure	352
	7.7.1	Preparatory calculations	352
	7.7.2	Galerkin approximation	353

0.2 Reportableed aductions of the continuity equilition

	7.7.3	Local existence of solutions	354			
	7.7.4	Existence of maximal solutions	357			
	7.7.5	Energy inequalities and estimates	360			
7.8	Complete system with dissipation in the continuity					
	equat	ion and with artificial pressure	361			
	7.8.1	Limit in the modified continuity equation	362			
	7.8.2	Limit in the momentum equation	363			
	7.8.3	Limit in the energy inequality and estimates				
		independent of vanishing dissipation	365			
	7.8.4	Improved estimate of density	366			
7.9	Comp	olete system with artificial pressure	368			
	7.9.1	Weak limits as dissipation tends to zero	369			
	7.9.2	Effective viscous flux	372			
	7.9.3	Renormalized equation of continuity and strong				
		convergence of density	374			
	7.9.4	Energy inequality and estimates independent of				
		artificial pressure	376			
	7.9.5	Improved estimate of density	376			
7.10	Comp	olete system of isentropic Navier–Stokes equations	381			
	7.10.1	Weak limits at vanishing artificial pressure	382			
	7.10.2	Effective viscous flux	386			
	7.10.3	Amplitude of oscillations	386			
	7.10.4	Renormalized continuity equation	388			
	7.10.5	Strong convergence of the density	390			
	7.10.6	Energy inequalities	392			
	7.10.7	General initial conditions	392			
7.11	Existe	ence of solutions in exterior domains	393			
	7.11.1	Solutions on invading domains	393			
	7.11.2	Orlicz spaces $L^p_a(\Omega)$	395			
	7.11.3	Estimates independent of invading domains	396			
	7.11.4	Improved estimates of density	397			
	7.11.5	Weak limits at growing invading domains	398			
	7.11.6	Effective viscous flux and renormalized				
		continuity equation	400			
indiana	7.11.7	Strong convergence of the density	401			
	7.11.8	Energy inequality	404			
7.12	Other	problems and bibliographic remarks	404			
	7.12.1	Bibliographic remarks on basic theorems	404			
	7.12.2	Slip boundary conditions	408			
	7.12.3	Nonmonotone pressure	409			
	7.12.4	Domain dependence	410			
	7.12.5	Nonhomogeneous boundary conditions	412			
	7.12.6	Unbounded domains and non-zero velocity at				
		infinity	424			

2

		7.12.7 Domains with nonsmooth boundaries	429
8	Glo	bal behavior of weak solutions	431
	8.1	Formulation of the problem	431
	8.2	Basic assumptions	432
	8.3	Sequential stabilization of the weak solution	432
	8.4	Auxiliary functions	433
	8.5	Existence and estimates of auxiliary functions	435
	8.6	Comparison density and a test function	437
	8.7	Passing to the limit with the regularization parameter	437
	8.8	Comparison density is close to the density as $t \to \infty$.	438
	8.9	Convergence of the density	446
	8.10	Uniqueness of equilibrium	452
	8.11	Global behavior of weak solutions in time in bounded	
		domains – arbitrary forces	456
	8.12	Bounded absorbing sets	457
	8.13	Asymptotically closed trajectories	458
	8.14	Global attractor of short trajectories	459
	8.15	Rapidly oscillating external forces	460
	8.16	Attractors	460
	8.17	Time-periodic solutions	461
	8.18	Uniqueness of equilibrium revisited	462
9	Stro	ong solutions of nonsteady compressible	
	Nav	ier–Stokes equations	464
	9.1	Problem formulation	464
	9.2	Similarity transformation	465
	9.3	Maximal parabolic regularity	466
	9.4	Resolution of the continuity equation with a given	
		velocity	467
	9.5	Further transcription of the problem	469
	9.6	Fixed point argument and the existence of a local	
		colution	170

Solution 410 9.7 Uniqueness 2.11.6. Eller has the share has she reported at 1. 473 9.8 Global a priori estimate 474 Global existence 9.9 479 **Bibliographical remarks** 480 9.10 References 485 Calculus opposition of choose and an analysis of a second of a second seco

in the conditionally equilation with the provident provident of the second second

499

Index