
Contents

Preface	ix
List of Figures	xi
List of Tables	xiii
1 Statistical Models	1
1.1 Models and parametrizations	1
1.1.1 Examples of statistical models	2
1.1.2 Reparametrization	5
1.1.3 Parameter functions	7
1.1.4 Nuisance parameters and parameters of interest	8
1.2 Likelihood, score, and information	8
1.2.1 The likelihood function	8
1.2.1.1 Formal definition	8
1.2.1.2 Equivariance of the likelihood function	11
1.2.1.3 Likelihood as a random variable	11
1.2.2 Score and information	12
1.2.3 Reparametrization and repetition	17
1.3 Exercises	22
2 Linear Normal Models	25
2.1 The multivariate normal distribution	25
2.2 The normal distribution on a vector space	30
2.2.1 Random vectors in V	30
2.2.2 Projections with respect to the concentration	34
2.2.3 Derived distributions	36
2.3 The linear normal model	40
2.3.1 Basic structure	40
2.3.2 Likelihood, score, and information	43
2.4 Exercises	44
3 Exponential Families	47
3.1 Regular exponential families	47
3.2 Examples of exponential families	48
3.3 Properties of exponential families	52

3.4	Constructing exponential families	53
3.4.1	Product families	53
3.4.2	Repeated observations	54
3.4.3	Transformations	55
3.4.4	Affine subfamilies	56
3.5	Moments, score, and information	59
3.6	Curved exponential families	62
3.7	Exercises	68
4	Estimation	73
4.1	General concepts and exact properties	73
4.2	Various estimation methods	82
4.2.1	The method of least absolute deviations	83
4.2.2	The method of least squares	84
4.2.3	M-estimators	87
4.2.4	The method of moments	87
4.3	The method of maximum likelihood	90
4.3.1	General considerations	90
4.3.2	Maximum likelihood in regular exponential families	94
4.4	Exercises	96
5	Asymptotic Theory	101
5.1	Asymptotic consistency and normality	102
5.2	Asymptotics of moment estimators	103
5.3	Asymptotics in regular exponential families	106
5.3.1	Asymptotic consistency of maximum likelihood	106
5.3.2	Asymptotic normality of maximum likelihood	107
5.3.3	Likelihood ratios and quadratic forms	111
5.4	Asymptotics in curved exponential families	116
5.4.1	Consistency of the maximum-likelihood estimator	116
5.4.2	Asymptotic normality in curved exponential families	120
5.4.3	Geometric interpretation of the score equation	121
5.4.4	Likelihood ratios and quadratic forms	123
5.5	More about asymptotics	128
5.6	Exercises	130
6	Set Estimation	135
6.1	Basic issues and definition	135
6.2	Exact confidence regions by pivots	135
6.3	Likelihood-based regions	139
6.4	Confidence regions by asymptotic pivots	140
6.4.1	Asymptotic likelihood-based regions	141
6.4.2	Quadratic score regions	142
6.4.3	Wald regions	143
6.4.4	Confidence regions for parameter functions	145

CONTENTS

vii

6.5	Properties of set estimators	148
6.5.1	Reparametrization	148
6.5.2	Coverage and length	150
6.6	Credibility regions	151
6.7	Exercises	153
7	Significance Testing	157
7.1	The problem	157
7.2	Hypotheses and test statistics	157
7.2.1	Formal concepts	157
7.2.2	Classifying hypotheses by purpose	158
7.2.3	Mathematical classification of hypotheses	159
7.3	Significance and p -values	159
7.4	Critical regions, power, and error types	161
7.5	Set estimation and testing	163
7.6	Test in linear normal models	164
7.6.1	The general case	164
7.6.1.1	Linear hypothesis, variance known	164
7.6.1.2	Linear subhypothesis, variance unknown	165
7.6.2	Some standard tests	166
7.6.2.1	Z -test for a given mean, variance known	166
7.6.2.2	T -test for a given mean, variance unknown	166
7.6.2.3	T -test for comparing means	167
7.6.2.4	T -test for paired comparisons	168
7.7	Determining p -values	168
7.7.1	Monte Carlo p -values	169
7.7.1.1	Simple hypotheses	169
7.7.1.2	Composite hypotheses	169
7.7.2	Asymptotic p -values	169
7.7.2.1	Simple hypotheses	169
7.7.2.2	Composite hypotheses	171
7.7.2.3	Smooth hypotheses	173
7.8	Exercises	175
8	Models for Tables of Counts	179
8.1	Multinomial exponential families	179
8.1.1	The unrestricted multinomial family	179
8.1.2	Curved multinomial families	181
8.1.2.1	Score and information	182
8.1.2.2	Likelihood ratio	182
8.1.2.3	Wald statistics	182
8.1.3	Residuals	183
8.1.4	Weldon's dice	184
8.2	Genetic equilibrium models	186

8.2.1	Hardy–Weinberg equilibrium	186
8.2.2	The ABO blood type system	188
8.3	Contingency tables	189
8.3.1	Comparing multinomial distributions	189
8.3.1.1	Comparing two multinomial distributions	189
8.3.1.2	Comparing two proportions	191
8.3.1.3	Comparing several multinomial distributions	195
8.3.2	Independence of classification criteria	196
8.3.3	Poisson models for contingency tables	198
8.3.3.1	The simple multiplicative Poisson model	199
8.3.3.2	The shifted multiplicative Poisson model	203
8.3.4	Sampling models for tables of counts	207
8.3.4.1	From multiplicative Poisson to independence	207
8.3.4.2	From independence to homogeneity	208
8.3.4.3	Exact conditional tests	208
8.3.5	Fisher’s exact test	210
8.3.6	Rank statistics for ordinal data	212
8.4	Exercises	213
A	Auxiliary Results	217
A.1	Euclidean vector spaces	217
A.2	Convergence of random variables	220
A.2.1	Convergence in probability	220
A.2.2	Convergence in distribution	221
A.2.3	The delta method	223
A.3	Results from real analysis	224
A.3.1	Inverse and implicit functions	224
A.3.2	Taylor approximation	226
A.4	The information inequality	227
A.5	Trace of a matrix	228
B	Technical Proofs	229
B.1	Analytic properties of exponential families	229
B.1.1	Integrability of derivatives	229
B.1.2	Quadratic regularity of exponential families	230
B.2	Asymptotic existence of the MLE	231
B.3	Iterative proportional scaling	233
Bibliography		237
Index		239