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We explain succinctly the purpose of each chapter:

1. Getting into the spirit of the course.
2. Writing about sets and functions.
3. Writing about sequences and equations.
4. The logic underpinning complex sentences.
5. Advanced writing on functions.
6. Improving various aspects of a mathematical text.
7. Structuring a mathematical proof.
8. Using induction arguments.
9. Writing definitions.
10. Writing a thesis.

Chapter 1 should be read by everyone. Chapters 2 and 3 are fundamental but introductory; an advanced student should be familiar with much of their content, and will consult them only if necessary. Chapter 4 develops the basic logic needed,