

Contents

Introduction	ix
Chapter 1. Order and residuation	1
1.1. Partially ordered sets and lattices	1
1.2. Residuated maps	6
1.3. Closure and co-closure operators	9
1.4. Residuated lattices: the algebras of logic	12
1.5. Nuclei and co-nuclei	19
1.6. Historical excursus	24
1.7. Bibliographical remarks	32
Chapter 2. Proof systems	35
2.1. Rules and derivations	35
2.2. A proof system for lattices	37
2.3. The full Lambek calculus	41
2.4. Adding structural rules	47
2.5. Hypersequent calculi	55
2.6. Historical excursus	60
2.7. Bibliographical remarks	69
Chapter 3. Consequence relations	71
3.1. Abstract consequence relations	71
3.2. Equational consequence relations	75
3.3. Equivalence of consequence relations	79
3.4. Residuated lattices and the full Lambek calculus	84
3.5. Historical excursus	86
3.6. Bibliographical remarks	93
Chapter 4. Structure theory	95
4.1. Convex subuniverses	95
4.2. Polars and prime convex subuniverses	99
4.3. Congruence relations	105
4.4. Normal convex subuniverse generation	110
4.5. Bibliographical remarks	113
Chapter 5. Semilinearity and distributivity	115
5.1. Equational bases for semilinear varieties	115
5.2. Densifiable varieties	119
5.3. Representations of distributive varieties	128
5.4. Generation and decidability results	131
5.5. Bibliographical remarks	136

Chapter 6. Cancellativity	139
6.1. Cancellative residuated lattices	139
6.2. Lattice-ordered groups of left quotients	144
6.3. A categorical equivalence	147
6.4. Bibliographical remarks	151
Chapter 7. Divisibility	153
7.1. GBL-algebras and GMV-algebras	153
7.2. Direct decomposition	158
7.3. Ordinal decomposition	161
7.4. Cone algebras and negative cones	165
7.5. A categorical equivalence	171
7.6. Strongly simple GBL-algebras	180
7.7. Bibliographical remarks	182
Chapter 8. Bridges between algebra and logic	185
8.1. The amalgamation property	185
8.2. The congruence extension property	192
8.3. Interpolation properties	195
8.4. Amalgamation in varieties of residuated lattices	199
8.5. Bibliographical remarks	204
Chapter 9. Finite embeddings and finite models	207
9.1. The finite embeddability property	207
9.2. Finite model properties	214
9.3. Join-extensions and join-completions	219
9.4. The FEP for varieties of residuated lattices	224
9.5. Bibliographical remarks	228
Appendix A. Open problems	231
Appendix B. Basic notions of universal algebra	237
Index	249
Bibliography	253