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**Foundations of Spline Theory: B-Splines, Spline Approximation, and Hierarchical Refinement**  
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**Abstract** This chapter presents an overview of polynomial spline theory, with special emphasis on the B-spline representation, spline approximation properties, and hierarchical spline refinement. We start with the definition of B-splines by means of

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Splines, in the broad sense of the term, are functions consisting of pieces of smooth functions glued together in a certain smooth way. Besides their theoretical interest, they have application in several branches of the sciences including geometric modeling, signal processing, data analysis, visualization, numerical simulation, and probability, just to mention a few. There is a large variety of spline species, often referred to as the zoo of splines. The most popular species is the one where the pieces are algebraic polynomials and inter-smoothness is imposed by means of equality of