Contents

1 Introduction 1

2 Least Squares Orthogonal Distance Fitting 3
2.1 Definition of Error Function 3
2.2 Character of Solution 4
2.3 Optimization of $F(\mathbf{a}, u)$ 5
References 11

3 General Properties of Splines 13
References 17

4 ODF Using a Cubic Bézier 19
4.1 Fitting a Function with a Double Inflection Point 20
4.2 Initializing a Cubic Bézier 21
4.3 Optimizing the Fit 22
4.4 Continuity of the rms Error 24
4.5 Character of Coalescing Solutions 24
References 26

5 Topology of Merges/Crossovers 27
5.1 Center of Mass Fit 27
5.2 Example of Two Types of Merge/Crossover 30
5.3 Response to Change in g(t) 32
5.4 Distinguishing Between Type 1 and Type 2 Events 34
References 36

6 ODF Using a 5-Point B-Spline 37
6.1 Initializing a 5-Point B-Spline 38
6.2 Basis Functions of a 5-Point B-Spline 38
6.3 Decomposition into Two Bézier Segments 39
6.4 ODF Results for a 5-Point B-Spline 41

7 ODF Using a 6-Point B-Spline 43
7.1 Initializing a 6-Point B-Spline 43
7.2 Basis Functions of a 6-Point B-Spline 44
7.3 Decomposition into Three Bézier Segments 45
7.4 ODF Results for a 6-Point B-Spline 46

8 ODF Using a Quartic Bézier 49
8.1 Initializing a Quartic Bézier 49
8.2 ODF Results for a Quartic Bézier 50
8.3 Enumeration of Solutions 52
8.4 Abnormal Truncation of Solutions 54
8.5 Topological Comparison of Béziers and B-Splines 55
References 56

9 ODF Using a Beta2-Spline 57
9.1 Rendering a Beta2-Spline 58
9.2 Nonlinear Effects Due to ψ 61
9.3 Continuity of the Beta2-Spline Integrands 62
9.4 ODF Results for a Beta2-Spline 62
References 65

10 ODF Using a Beta1-Spline 67
10.1 Rendering a Beta1-Spline 67
10.2 Nonlinear Effects Due to β_1 68
10.3 Continuity of the Beta1-Spline Integrands 69
10.4 ODF Results for a Beta1-Spline 71
10.5 Numerical Problems with the Beta1-Spline 73

11 Conclusions 77