

Contents

<i>Preface</i>	<i>xiii</i>
<i>Acknowledgments</i>	<i>xiv</i>
<i>Organization of the book</i>	<i>xvi</i>
1. Introduction	1
1.1. Motivation: Newton vs. Einstein	2
1.2. General relativity: a new theory of gravity	3
1.3. Book outline	4
2. Special relativity	7
2.1. Newtonian (non-relativistic) physics	7
2.1.1. Inertial frames	7
2.1.2. Galilean transformations	8
2.1.3. Crisis of Newtonian physics	10
2.2. Foundational premises of special relativity	10
2.3. Lorentz transformations	11
2.3.1. Standard configuration	11
2.3.2. Composition of velocities and general Lorentz transformations	13
2.4. Length contraction	13
2.5. Time dilation	14
2.6. Invariance of the volume element	14
2.7. Timelike, spacelike, and null intervals	15
2.8. Recovering Newtonian mechanics: special limit $v \ll c$	16
2.9. Covariant and contravariant four-vectors	17
2.9.1. Four-velocity	19
2.10. Energy–momentum four-vector	20
2.11. Spacetime diagrams	22
2.12. Geodesics: equations of motion from an action principle	25
2.13. What an observer observes	26
3. The Equivalence Principle	31
3.1. Newtonian gravity is inconsistent with special relativity	31
3.2. The equivalence of gravitational and inertial mass	31
3.2.1. Light must also feel gravity	33
3.2.2. Clocks in a uniform gravitational field	33
3.3. A scientific fable	35
3.4. Gravity as geometry	37

3.5.	Geometric derivation of Newtonian gravity	38
4.	General relativity	41
4.1.	Notation and conventions	41
4.2.	General coordinate transformation	42
4.3.	Spacetime coordinates and their metric tensors	44
4.3.1.	Symmetries of a metric tensor	46
4.4.	Metric tensors	47
4.4.1.	Flat Euclidean space	47
4.4.2.	Flat Minkowski spacetime	47
4.4.3.	Curved spacetime	48
4.5.	Freely falling frames	49
4.6.	Area, volume, and four-volume	51
4.7.	Vectors in curved spacetime	52
4.7.1.	Coordinate bases	53
4.7.2.	Orthonormal bases	54
4.8.	Covariant derivative	57
4.9.	Geodesic equation	58
4.9.1.	Recovering Newtonian gravity	61
5.	Einstein's field equations	65
5.1.	Bulk macroscopic description of matter: the Eulerian form	65
5.1.1.	Conservation laws in flat spacetime	67
5.2.	The energy–momentum tensor	68
5.3.	The Riemann tensor, the Ricci tensor, the Ricci scalar, and the Einstein tensor	70
5.4.	Evolution of energy density	72
5.4.1.	Matter density	74
5.4.2.	Radiation density	74
5.5.	Einstein's field equations	74
6.	The Schwarzschild metric and black holes	77
6.1.	The Schwarzschild problem	77
6.2.	Solving the Schwarzschild problem	78
6.3.	Schwarzschild radius, event horizon, and black holes	83
6.4.	Orbits in Schwarzschild's geometry	84
7.	The cosmological metric and Friedmann's equations	87
7.1.	Hubble's law and the evolving Universe	87
7.2.	Friedmann's equations	91
7.3.	A Newtonian analogy to Friedmann's equations	94

8. Solutions of Friedmann's equations	97
8.1. Cosmological models	97
8.1.1. Equation of state for the radiation-dominated Universe: perfect fluid	97
8.1.2. Equation of state for the matter-dominated Universe: dust	98
8.1.3. Curvature of the Universe	99
8.2. Flat Universe ($k = 0, q_0 = 1/2$)	100
8.2.1. Radiation-dominated Universe (perfect-fluid approximation)	100
8.2.2. Matter-dominated Universe (dust approximation)	101
8.3. Closed Universe ($k = +1, q_0 > 1/2$)	102
8.3.1. Radiation-dominated Universe (perfect-fluid approximation)	102
8.3.2. Matter-dominated Universe (dust approximation)	103
8.4. Open Universe ($k = -1, q_0 < 1/2$)	105
8.4.1. Radiation-dominated Universe (perfect-fluid approximation)	105
8.4.2. Matter-dominated Universe (dust approximation)	106
8.5. Big Bang: a radiation/matter-dominated singularity	107
9. Cosmological constant and the dark Universe	111
9.1. Age of a matter-dominated Friedmann Universe	111
9.2. Einstein's field equations revisited: cosmological constant	112
9.3. Age of the Universe revisited	115
9.4. Particle horizon revisited: the horizon problem	116
9.5. Possible solutions to the horizon problem	117
9.5.1. Inflation	117
10. Cosmic distances	123
10.1. Redshift	123
10.2. Proper distance	124
10.3. Comoving coordinates and comoving distance	124
10.4. Angular diameter distance	126
10.5. Luminosity distance	127
10.6. Particle horizon	128
11. Summary of the foundations of cosmology	133
11.1. General relativity: kinematics in curved spacetime	133
11.2. Cosmology: solutions of Friedmann's equations	135
11.3. Expanding Universe	136
12. Cosmic content	139
12.1. Particle distribution function	139
12.2. Particle distribution function of species	141
12.3. Entropy density	142

12.4. Cosmic microwave background photons	143
12.5. Cosmic neutrino background	144
12.6. Baryonic matter	148
12.7. Dark matter	148
12.7.1. Evidence for dark matter: rotation curves in disk galaxies	150
12.7.2. Other evidence for dark matter	151
12.7.3. Dark-matter candidates	152
12.8. Dark energy	156
12.8.1. Detecting dark energy using luminosity distance vs. redshift	158
12.8.2. Investigating other forms of dark energy	161
12.8.3. Alternative to dark energy	162
13. Brief history of the early Universe	165
13.1. Keeping track of the Universe's history	165
13.2. Early Universe at a glance: major benchmarks	166
13.3. <i>Very</i> early Universe: the first three minutes	167
13.3.1. The Big Bang: $t = 0$ s	167
13.3.2. The Planck epoch: $0 < t \lesssim 10^{-43}$ s	168
13.3.3. Grand unification epoch: 10^{-43} s $\lesssim t \lesssim 10^{-36}$ s	169
13.3.4. Inflationary epoch: 10^{-36} s $\lesssim t \lesssim 10^{-32}$ s	169
13.3.5. Electroweak epoch: 10^{-32} s $\lesssim t \lesssim 10^{-12}$ s	170
13.3.6. Quark epoch: 10^{-12} s $\lesssim t \lesssim 10^{-6}$ s	170
13.3.7. Hadron epoch: 10^{-6} s $\lesssim t \lesssim 1$ s	171
13.3.8. Lepton epoch: 1 s $\lesssim t \lesssim 3$ min	171
13.4. Early Universe: the first 400,000 years	173
13.4.1. The earliest Universe: $T \gtrsim 10^{12}$ K	173
13.4.2. Neutrino decoupling: $T \approx 3 \times 10^{10}$ K	174
13.4.3. Neutron:proton ratio freezes out: $T \approx 10^{10}$ K	175
13.4.4. Electron–positron annihilation: $T \approx 3 \times 10^9$ K	176
13.4.5. The epoch of nucleosynthesis: 10^9 K $\gtrsim T \gtrsim 3 \times 10^8$ K	176
13.4.6. Transition from radiation-dominated Universe to matter-dominated Universe: $T \approx 10,000$ K	176
13.4.7. Recombination: $T \approx 4000$ K	177
14. Cosmic microwave background radiation	181
14.1. Importance of the CMB radiation	181
14.1.1. Blackbody spectrum	182
14.1.2. Number density of the CMB photons	183
14.2. Systematic bias: the dipole anisotropy	184
14.2.1. Relativistic Doppler shift	185
14.3. Angular power spectrum	187
14.3.1. Two-point correlation function vs. angular power spectrum	189
14.3.2. Cosmic variance	189

14.4. Scales in the angular power spectrum	190
14.5. Baryon acoustic oscillations	190
14.5.1. Dampening of the overtones	192
14.6. Physical effects affecting the CMB radiation	193
14.6.1. Sunyaev–Zel’dovich effect	193
14.6.2. Sachs–Wolfe effect	193
14.6.3. Integrated Sachs–Wolfe effect	194
<i>Epilogue</i>	197
<i>Further reading</i>	199
A. An alternative Lagrangian	201
B. Geodesic equation in spherical coordinates	203
C. Example of metric conversion	207
D. Applying the geodesic equation	209
E. Matter–dark energy equality	211
F. Radiation–dark energy equality	213
G. Radiation–matter equality	215
H. Chemical potential	217
I. How to compute the relative abundances of the light elements	219
J. Equation of state for the perfect fluid	223
K. Origins of the large-scale structure in the Universe	225
<i>Constants, units, and conversions</i>	231
<i>References</i>	233
<i>Index</i>	237