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Most books dealing with these subjects follow either a semi-classical approach, where the field is treated as a classical field interacting with quantum particles, or a full quantum approach where both systems are quantized. The first approach is often oversimplified and fails to describe correctly new situations that can now be investigated with the development of sophisticated experimental techniques. The second approach is often too difficult for beginners and lacks simple physical pictures, very useful for an initial understanding of a physical phenomenon. The advantage of this book is that it gives both approaches, starting with the first, illustrated by several simple examples, and introducing progressively the second, clearly showing why it is essential for the understanding of certain phenomena. The authors also clearly demonstrate, in the case of non-linear optics and laser cooling, how advantageous it may be to combine both approaches in the analysis of an experimental situation and how one can get from each point of view useful, complementary physical insights. I believe that this challenge to present and to illustrate both approaches in a single book has been taken up successfully. Whatever their ultimate interests, the readers of this work will be exposed to an important example of a lived and vibrant field of research and they will better understand the intellectual stimulation and the technical developments which result from it.

To write a book on such a broad topic, the authors must obviously possess wide knowledge of the field, they must have thought long and hard about the basic concepts and about the different levels of complexity with which one can approach the topics. They must also have a deep and concrete knowledge about experimental and technical details and the many problems which daily confront a laboratory researcher. Having worked extensively with them, I know the authors of this work fulfil these requirements. I have the highest admiration for their conviction, their scientific rigour, their ability to give simple and precise physical explanations, and their quest to illuminate clearly the difficult points of the subject without oversimplification. Each of them has made many original contributions to the development of this important field of physics, and they and their younger collaborators are this book's work with the cutting edge of modern quantum optics. In reading the book, I am therefore not surprised to find their many fine qualities reflected in the text. The general